PEARSON

MATHEMATICAL METHODS

QUEENSLAND

STUDENT BOOK

11

UNITS 1 & 2

Contents

UNIT 2

Supporting the integrating of technology

Students are supported with the integration of technology in a number of ways. The eBook includes 'How to' user guides covering all basic functionality for the following three graphing calculators:

- TI-84 Plus CE
- TI-Nspire CX (non CAS)
- CASIO fx-CG50AU

Throughout the student book are Technology worked examples strategically placed within the theory. These are suitable for both the TI-Nspire CX (non CAS) and CASIO fx-CG50AU. The examples clearly demonstrate how the technology can be used effectively and efficiently for the content covered in that chapter.

Graphing calculators are not the only technology integrated throughout the Pearson Queensland senior mathematics series. Spreadsheets, Desmos and interactive widgets have been included to provide students with the opportunity to visualise concepts, consolidate their understanding and make mathematical connections.

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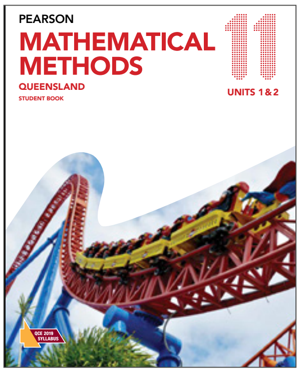
MATHEMATICAL METHODS 11

QUEENSLAND

UNITS 1 & 2



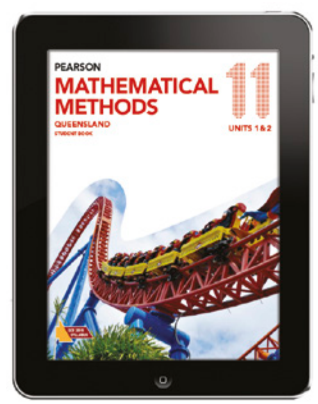
Mathematical Methods 11
Student book

Student book

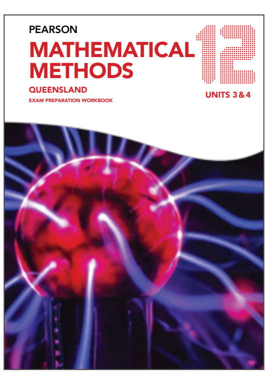
The student book has been written by local authors, ensuring quality content and complete curriculum coverage for Queensland, enabling students to prepare with ease and confidence. We have covered the breadth of the content within our exercise questions, from simpler skills-focused questions to those using unfamiliar contexts and application of the theory learnt. The theory, worked examples and question sets are written in line with the assessment objectives, with the aim of familiarising students with QCE cognitive verbs in the process of dependent and guided instruction. Additional interactives that help explain the theory and consolidate concepts have been included throughout all chapters.

Pearson Reader+

Pearson Reader+ is our next-generation eBook. This is an electronic textbook that students can access on any device, online or offline, and is linked to features, interactives and visual media that will help consolidate their understanding of concepts and ideas, as well as other useful content developed specifically for senior mathematics. It supports students with appropriate online resources and tools for every section of the student book, providing access to exemplar worked solutions that demonstrate high levels of mathematical and everyday communication. Students will have the opportunity to learn independently through the Explore further tasks and Making connections interactive widgets, which have been designed to engage and support conceptual understanding. Additionally, teachers have access to syllabus maps, a teaching program, sample exams, problem-solving and modelling tasks, and additional banks of questions for extra revision.



Mathematical Methods 11
eBook



Mathematical Methods 12
Exam preparation workbook

Exam preparation workbook

Additional component for Year 12 only

The exam preparation workbook provides additional support in preparing students for the external exam. It has been constructed to guide the students through a sequence of preparatory steps and build confidence leading up to the external exam.

How to use this book

Pearson Mathematical Methods 11 Queensland Units 1 & 2

This Queensland senior mathematics series has been written by a team of experienced Queensland teachers for the QCE 2019 syllabus. It offers complete curriculum coverage, rich content and comprehensive teacher support.

Key information

Key information and rules are highlighted throughout the chapter.

Warning boxes

Warning boxes are located throughout the chapter to alert students to common errors and misconceptions.

Making connections

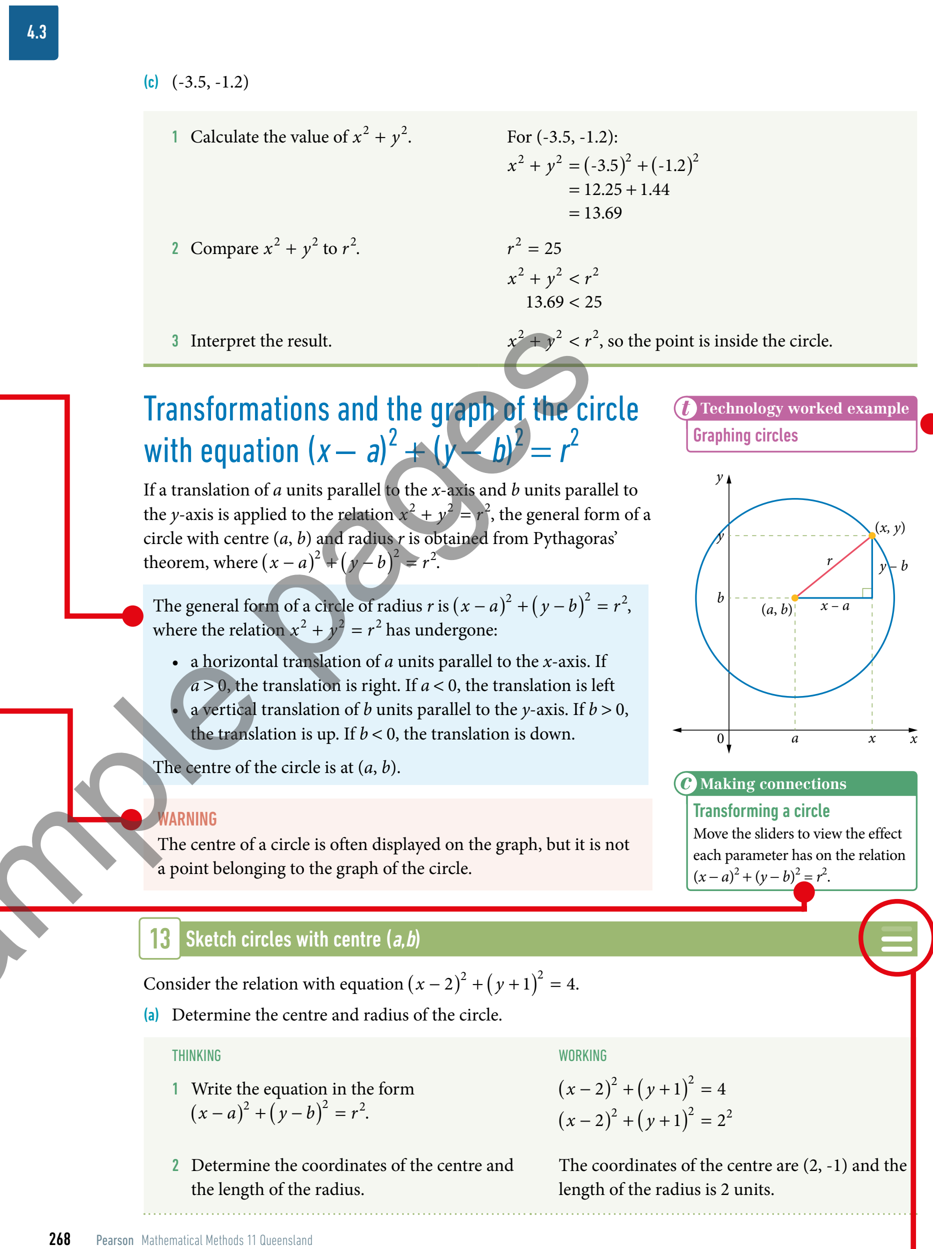
This eBook feature provides teachers and students with a visual interactive of specific mathematics concepts or ideas to aid students in their understanding.

Every worked example and question is graded

Every example and question is graded using the three levels of difficulty, as specified in the QCE syllabus:

- simple familiar (1 bar)
- complex familiar (2 bars)
- complex unfamiliar (3 bars)

The visibility of this grading helps ensure all levels of difficulty are well covered.



4.3

(c) (-3.5, -1.2)

1	Calculate the value of $x^2 + y^2$.	For (-3.5, -1.2): $x^2 + y^2 = (-3.5)^2 + (-1.2)^2 = 12.25 + 1.44 = 13.69$
2	Compare $x^2 + y^2$ to r^2.	$r^2 = 25$; $x^2 + y^2 < r^2$; $13.69 < 25$
3	Interpret the result.	$x^2 + y^2 < r^2$, so the point is inside the circle.

Transformations and the graph of the circle with equation $(x - a)^2 + (y - b)^2 = r^2$

If a translation of a units parallel to the x-axis and b units parallel to the y-axis is applied to the relation $x^2 + y^2 = r^2$, the general form of a circle with centre (a, b) and radius r is obtained from Pythagoras' theorem, where $(x - a)^2 + (y - b)^2 = r^2$.

The general form of a circle of radius r is $(x - a)^2 + (y - b)^2 = r^2$, where the relation $x^2 + y^2 = r^2$ has undergone:

- a horizontal translation of a units parallel to the x-axis. If $a > 0$, the translation is right. If $a < 0$, the translation is left
- a vertical translation of b units parallel to the y-axis. If $b > 0$, the translation is up. If $b < 0$, the translation is down.

The centre of the circle is at (a, b).

WARNING

The centre of a circle is often displayed on the graph, but it is not a point belonging to the graph of the circle.

Technology worked example

Graphing circles

Making connections

Transforming a circle

Move the sliders to view the effect each parameter has on the relation $(x - a)^2 + (y - b)^2 = r^2$.

13 Sketch circles with centre (a, b)

Consider the relation with equation $(x - 2)^2 + (y + 1)^2 = 4$.

(a) Determine the centre and radius of the circle.

	THINKING	WORKING
1	Write the equation in the form $(x - a)^2 + (y - b)^2 = r^2$.	$(x - 2)^2 + (y + 1)^2 = 4$; $(x - 2)^2 + (y + 1)^2 = 2^2$
2	Determine the coordinates of the centre and the length of the radius.	The coordinates of the centre are (2, -1) and the length of the radius is 2 units.

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Technology worked examples

These worked examples offer support in using technology such as spreadsheets, graphing calculators and graphing software, and include technology-focused worked examples and activities.

Meeting the needs of the QCE Syllabus

The authors have integrated both the cognitive verbs and the language of the syllabus objectives throughout the worked examples and questions.

(b) Determine the axes intercepts.

1 Determine the coordinates of the x-intercepts.

x-intercept, let $y = 0$:

$$(x-2)^2 + (1)^2 = 4$$
$$(x-2)^2 = 3$$
$$x - 2 = \pm\sqrt{3}$$
$$x = 2 \pm \sqrt{3}$$

The coordinates of the x-intercepts are $\left(2-\sqrt{3},0\right)$ and $\left(2+\sqrt{3},0\right)$.

2 Determine the coordinates of the y-intercept(s).

y-intercept, let $x = 0$:

$$(-2)^2 + (y+1)^2 = 4$$
$$4 + (y+1)^2 = 4$$
$$(y+1)^2 = 0$$
$$y + 1 = 0$$
$$y = -1$$

The coordinates of the y-intercept are (0, -1).

Tech-free questions to practice and maintain by-hand skills

Every exercise includes 'tech-free' questions to help students continuously practice their by-hand skills in preparation for Exam Paper 1.

6 Consider the cubic expressions in expanded form:

(a) For $-2x(x+11)(x-11)$ the number of terms will be:

A 1 B 2 C 3 D 4

(b) For $4(x-4)^3$ the number of terms will be:

A 1 B 2 C 3 D 4

(c) Explain the common error made by a student who chose the second incorrect option in part (b).

Worked solutions

Fully worked solutions are provided for every question in the student textbook and can be accessed from the accompanying eBook.

Highlighting common errors

Throughout the exercises, authors have integrated questions designed to highlight common errors frequently made by students. Explanations are given in the worked solutions.

Additional information

These interactives appear in the eBook in two forms, as videos explaining specific concepts or as interactive questions to check students' understanding.

Additional information

The vertex of a parabola

This activity explores how the coordinates of the turning points can be derived from the general quadratic expression.

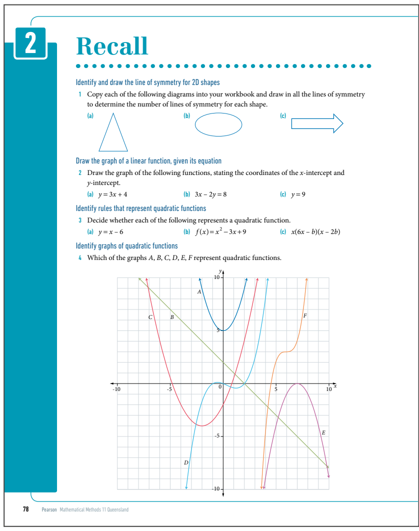
Explore further

This eBook feature provides an opportunity for students to consolidate their understanding of concepts and ideas with the aid of technology, and answer a small number of questions to deepen their understanding and broaden their skills base. These activities should take approximately 5–15 minutes to complete.

Explore further

Exploring cubic functions in the form $y = ax^3 + bx^2 + cx + d$

Explore the effect of varying the coefficients on the graph of a cubic polynomial expressed in the general form.

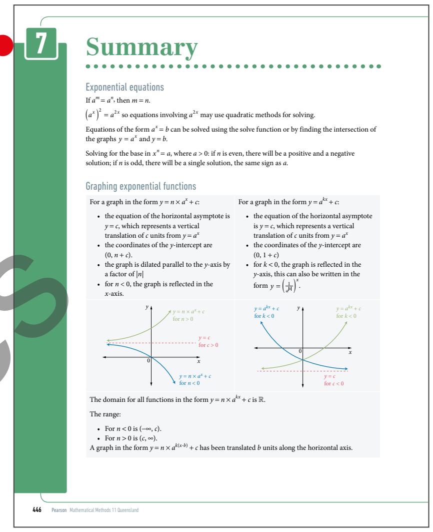


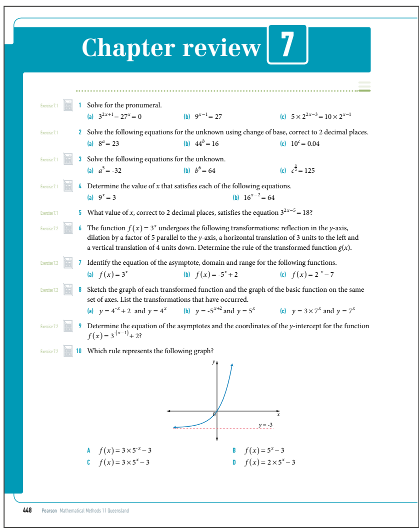
Recall

Each chapter begins with a review of assumed knowledge for the chapter.

Summary

At the end of each chapter, there is a summary of the key facts and rules used in the chapter.





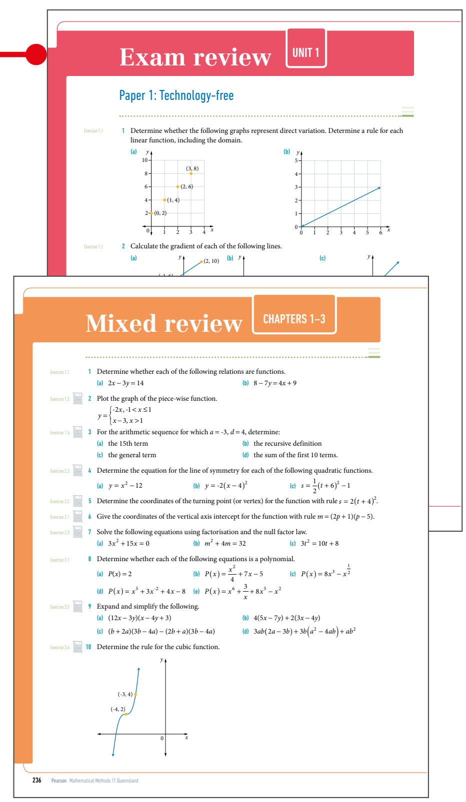
Chapter review

Every chapter review follows the QCAA examination proportions for levels of difficulty, which is 60% simple familiar, 20% complex familiar and 20% complex unfamiliar.

Mixed and Exam review

Exam reviews give cumulative practice of content already covered, to prepare students for the end-of-year exam. They have been placed at the end of each unit.

As well, Mixed reviews provide cumulative revision that mixes content from previous chapters placed midway through each unit.



4

Advanced functions and relations

Recall

Calculate values using algebra

1 For the rules below, determine the corresponding y-value for the given x-values.

(a) $y = \dfrac{1}{x+2}$ for $x = 0$

(b) $y = 2 + \sqrt{x+4}$ for $x = 5$

(c) $x^2 + y^2 = 25$ for $x = 3$

Simplify surds

2 Simplify each of the following surds.

(a) $\sqrt{18}$

(b) $\dfrac{\sqrt{125}}{5}$

(c) $2\sqrt{32}$

(d) $2\sqrt{5} - 5\sqrt{5}$

Rationalise the denominator

3 Simplify the following surds and rationalise the denominator.

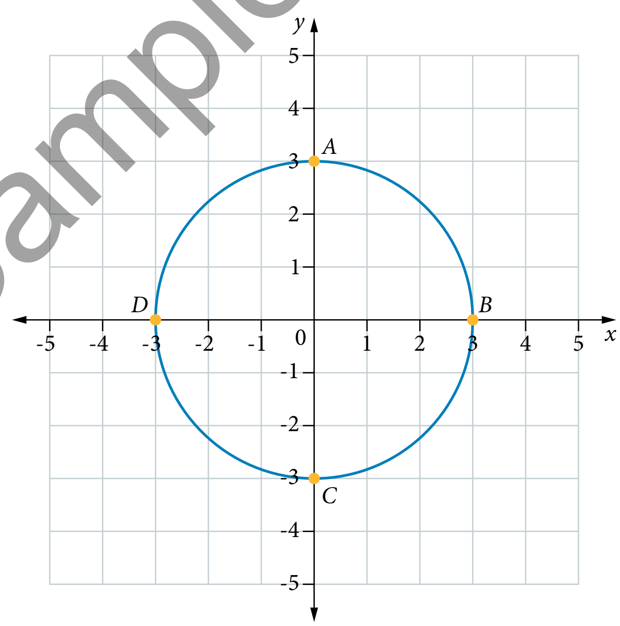
(a) $\dfrac{2}{\sqrt{5}}$

(b) $\dfrac{\sqrt{3}}{\sqrt{2}}$

(c) $\dfrac{3\sqrt{6}}{2\sqrt{7}}$

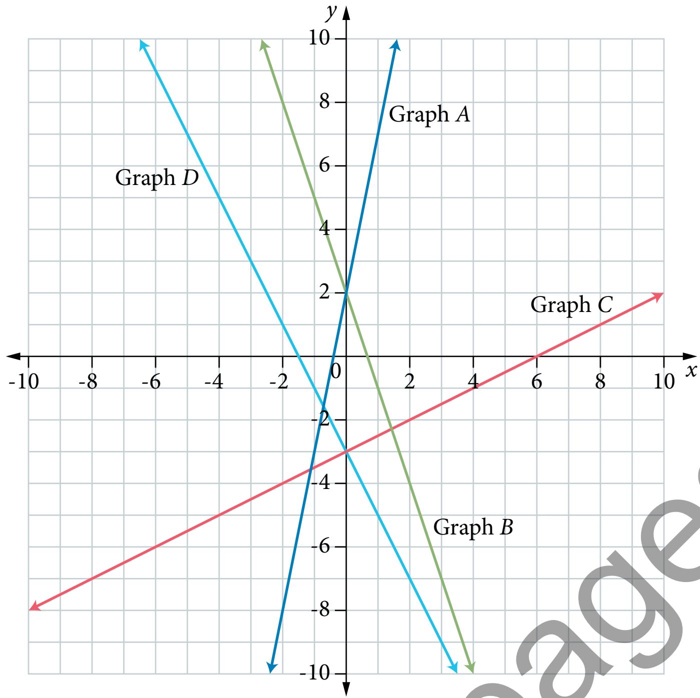
Label points on a graph

4 Determine the coordinates of the points A, B, C and D.



Determine the equations of lines from their graphs

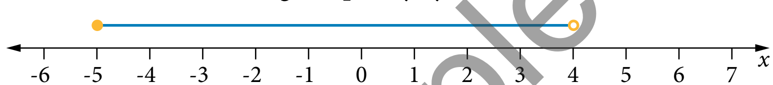
5 Determine the equation for each of the linear graphs, A, B, C and D, shown on the graph below.



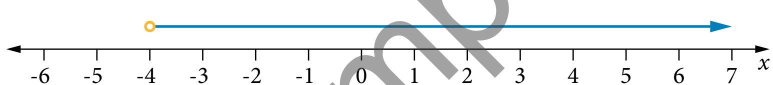
Label intervals of the real number line using set and interval notation

6 Describe each interval using inequality symbols and interval notation.

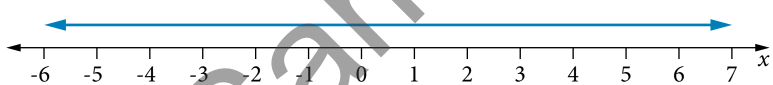
(a)



(b)



(c)



Complete a table of values

7 Complete the table of values for $y = \frac{1}{x}$.

x	-2	-1	$-\frac{1}{2}$	$\frac{1}{2}$	1	2
y						

Solve equations

8 Solve the following equations for x.

(a) $\frac{3x-4}{4} = 2$ (b) $\sqrt{x+4} = 1$ (c) $\frac{2}{x+4} = 1$

Determine the y-intercept of functions

9 Determine the coordinates of the y-intercept for each function.

(a) $y = 2x - 6$ (b) $y = \frac{1}{x-2} + 3.5$ (c) $y = \sqrt{x+1}$

4.1 Inverse proportion and graphs of rectangular hyperbolas

Inverse proportion

Inverse proportion is a situation in which an increase in one variable produces a decrease in another related variable. Examples of inverse proportion are:

- the further the distance a missile travels above the Earth, the less the force of gravity it experiences
- the more air that is blown into a balloon, the thinner the rubber becomes
- the further away from a rock band speaker, the softer the sound of the music
- the longer the time after being given an antibiotic injection, the less the chemical concentration in your bloodstream.

Two variables are inversely proportional if an increase in one variable produces a decrease in the other variable.

If y is inversely proportional to x, then $y = \frac{k}{x}$, where x and y are variables and k is a constant.

Additional information

Introduction to inverse proportion

Watch the video to further explore inverse proportion.

1 Average speed

A helicopter travels to a destination 60 km from its starting point. The rule $s = \frac{60}{t}$ can be used to determine the average speed of the helicopter s (km/h) for a trip of t (hours).

(a) Determine the speeds of the helicopter for trips that took:

(i) 30 minutes **(ii)** 1 hour **(iii)** 2 hours

THINKING

Substitute the time values into the equation $s = \frac{60}{t}$ to determine the average speed.

WORKING

(i) 30 minutes $= \frac{1}{2}$ hour

$$s = \frac{60}{t} = 60 \div \frac{1}{2} = 60 \times \frac{2}{1} = 120\text{ km/h}$$

(ii) 1 hour

$$s = \frac{60}{t} = \frac{60}{1} = 60\text{ km/h}$$

(iii) 2 hours

$$s = \frac{60}{t} = \frac{60}{2} = 30\text{ km/h}$$

(b) Describe how the average speed of the helipoter varies as the time for the trips changes.

Two variables are inversely proportional if an increase in one variable produces a decrease in the other variable.

As the time taken to complete the 60 km trip increases, the average speed decreases.

(c) If the average speed of the helicopter was 45 km/h, determine the time it took to complete the journey. Write your answer in hours and minutes.

1 Recall the formula $s = \frac{60}{t}$ and substitute the given speed. Solve for t, writing the time in hours and minutes.	$45 = \frac{60}{t}$ $t = \frac{60}{45}$ $t = 1\frac{1}{3}$ h $t = 1$ h 20 min
2 Interpret the result.	The helicopter took 1 h 20 min to complete the journey.

2 Inverse proportion and graphs

Four contestants in different events complete a time trial of a 120 km course. They travel with average speeds of 240 km h^{-1}, 120 km h^{-1}, 60 km h^{-1} and 30 km h^{-1} respectively for the journey. The equation for the time of travel t (hours) is $t = \frac{120}{s}$ and contestants travel at an average speed of s (km h^{-1})

(a) Complete the table of values for the time taken for the cars to complete the journey.

Speed (km h^{-1})	240	120	60	30
Time (hours)				

THINKING

Recall and transpose the formula $s = \frac{d}{t}$ to calculate the time taken for each car to complete the trial.

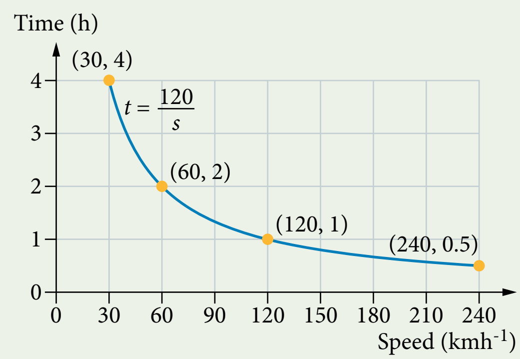
WORKING

$t = \frac{d}{s}$

Speed (km h^{-1})	240	120	60	30
Time (hours)	$\frac{120}{240} = 0.5$	$\frac{120}{120} = 1$	$\frac{120}{60} = 2$	$\frac{120}{30} = 4$

(b) Draw a graph of time versus speed.

Plot the points and join them with a smooth curve.

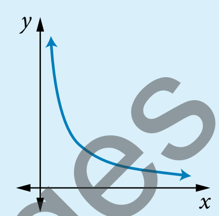


(c) Describe the shape of the graph.

Describe the shape of the graph.	The graph is a smooth decreasing curve: t increases as s decreases, and t decreases as s increases. The slower the average speed, the longer time it takes to complete the course, with the graph approaching the t-axis. The faster the average speed, the shorter the time it takes to complete the course, with the graph approaching the s-axis. The graph does not touch either of the axes.

The graph of $y = \frac{k}{x}$, where $k > 0$ is in the first quadrant only.

The graph approaches, but does not touch either axis.



You can determine a specific rule from the general form, as long as sufficient information is given. In the case of inverse proportion, with general rule $y = \frac{k}{x}$, where $x \neq 0$, one distinct point is sufficient to determine k.

Additional information

Graphs of inverse proportion

Explore additional explanations of graphs of inverse proportion.

3 Establish the inverse proportion rule, given a point that belongs to the function

The table on the right shows the time t (seconds) it takes for a standard tablet to dissolve in a fixed quantity of water at temperature T°C.

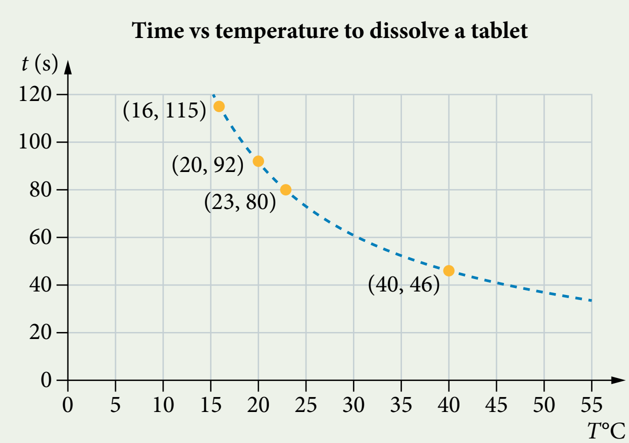
Plot the points and determine the rule of the function connecting T°C and t (s).

T°C	t (s)
16	115
20	92
23	80
40	46

THINKING

WORKING

1 Consider an appropriate scale for each axis and plot the points.



2 The general equation for inverse proportion is $y = \frac{k}{x}$, where $x \neq 0$. Identify the given information to determine the value of k.	Using (40, 46): $46 = \frac{k}{40}$ $k = 46 \times 40$ $= 1840$
3 Write the rule.	The rule is $t = \frac{1840}{T}$ where $t > 0$.

The rectangular hyperbola

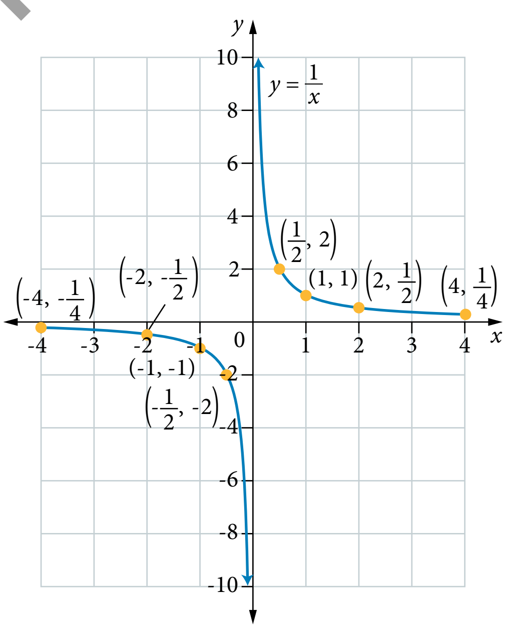
A graph showing inverse proportion is called a rectangular hyperbola. The simplest hyperbola is in the form $f(x) = \frac{1}{x}$, $x \neq 0$.

A table of values can display the coordinates on the graph, which can be used to explain its shape.

x	-4	-2	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1	2	4
y	$\frac{1}{-4} = -\frac{1}{4}$	$\frac{1}{-2} = -\frac{1}{2}$	$\frac{1}{-1} = -1$	$\frac{1}{-\frac{1}{2}} = -2$	$\frac{1}{0}$ is undefined	$\frac{1}{\frac{1}{2}} = 2$	$\frac{1}{1} = 1$	$\frac{1}{2}$	$\frac{1}{4}$

Examining the extreme values for x for $y = \frac{1}{x}$:

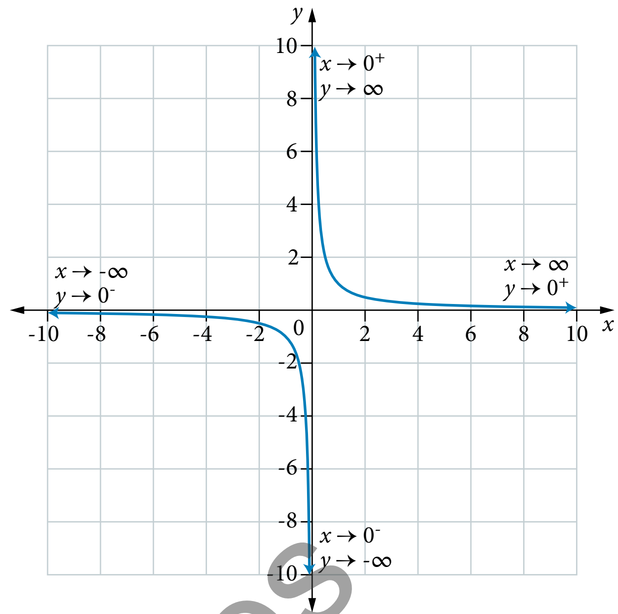
- As the x-values approach positive infinity, the y-values approach 0^+ (zero from above).
  As $x \to \infty$, then $y = \frac{1}{\infty}$ so $y \to 0^+$
- As the x-values approach negative infinity, the y-values approach 0^- (zero from below).
  As $x \to -\infty$, then $y = \frac{1}{-\infty}$ so $y \to 0^-$



Graphically, this means that the curve $y = \frac{1}{x}$ approaches the x-axis from above as $x \to \infty$ and from below as $x \to -\infty$, explaining the shape of the curve at these extremes. So the curve $y = \frac{1}{x}$ approaches but does not touch x-axis.

- As the x-values approach zero from the positive side, the y-values approach ∞.
  As $x \to 0^+$, then $y = \frac{1}{0^+}$ so $y \to \infty$
- As the x-values approach zero from the negative side, the y-values approach $-\infty$.
  As $x \to 0^-$, then $y = \frac{1}{0^-}$ so $y \to -\infty$.

Graphically, this means that as the curve $y = \frac{1}{x}$ approaches the value of $x = 0$ from above $y \to \infty$ and from below $y \to -\infty$, explaining the shape of the curve at these extremes. So the curve $y = \frac{1}{x}$ approaches but does not touch the y-axis.



An asymptote is a line that a curve approaches but doesn't touch. The graph of $y = \frac{1}{x}$ has two asymptotes: the x-axis, or the line with equation $y = 0$, and the y-axis, or the line with equation $x = 0$. In rectangular hyperbolas, the two asymptotes are at right angles.

The domain and the range of the function is $\mathbb{R} \setminus \{0\}$; all real numbers except for zero.

The shape of the curve for $f(x) = \frac{1}{x}, x \neq 0$, is the basic shape of a rectangular hyperbola that can be used to sketch other related curves using the transformations of dilation, reflection and translation.

Algebra is used to determine the axis intercepts to complete an accurate sketch of the function.

Making connections

Transforming a rectangular hyperbola

Move the sliders to view the effect of each parameter in the rule $f(x) = \dfrac{a}{(x-b)} + c$.

The general rule of a rectangular hyperbola is $f(x) = \dfrac{a}{(x-b)} + c$:

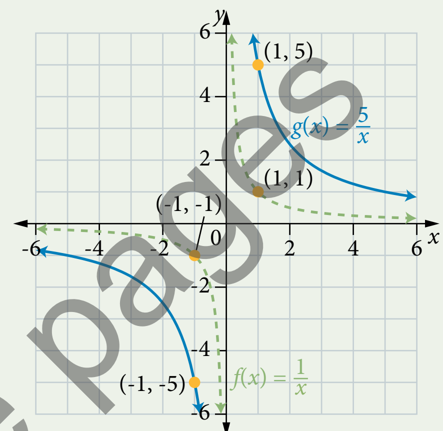
- $|a|$ dilates the curve parallel to the y-axis (or from the x-axis).
- $a < 0$ reflects the curve in the x-axis.
- b translates the graph in the x-direction (horizontally). The vertical asymptote is $x = b$.
- c translates the graph in the y-direction (vertically). The horizontal asymptote is $y = c$.
- The domain is $\mathbb{R} \setminus \{b\}$.
- The range is $\mathbb{R} \setminus \{c\}$.

4 Sketch rectangular hyperbolas

Sketch the graphs of $f(x) = \dfrac{1}{x}$ and each of the following on the same set of axes.

(a) $g(x) = \dfrac{5}{x}$

THINKING	WORKING
1 Write $g(x)$ in the form $f(x) = \dfrac{a}{x}$ and identify the values of a.	$g(x) = \dfrac{5}{x}$ $a = 5$
2 Identify any dilations or translations.	For $a = 5$, the graph of $f(x) = \frac{1}{x}$ has been dilated by a factor of 5 parallel to the y-axis, so the shape of $g(x)$ is steeper than $f(x)$.

3 Determine the equations of the asymptotes.	There are no translations, so the equations of the asymptotes are $x = 0$ and $y = 0$.
4 Check the reasonableness of the asymptotes.	An asymptote occurs where the denominator is 0: $x = 0$ An asymptote occurs where the fraction is equal to 0: $y = 0$.
5 A hyperbola in the form $f(x) = \frac{a}{x}$ will have no axes intercepts. Therefore, determine the coordinates of a point on each arm.	For $x = 1$: $g(1) = \frac{5}{1}$ $= 5$ Quadrant 1: (1, 5) For $x = -1$: $g(-1) = \frac{5}{-1}$ $= -5$ Quadrant 3: (-1, -5)
6 Sketch the graphs of $f(x)$ and $g(x)$, showing the major features.	

(b) $g(x) = \dfrac{-2}{x-3}$

1 Write $g(x)$ in the form $f(x) = \frac{a}{(x-b)}$ and identify the values for a and b.	$g(x) = \dfrac{-2}{x-3}$ $a = -2$ and $b = 3$
2 Identify any dilations or translations.	$a < 0$ represents a reflection in the x-axis. For $a = 2$, the dilation factor is 2 parallel to the y-axis, so the shape of $g(x)$ is steeper than $f(x)$. For $b = 3$, the graph of $g(x)$ is translated 3units to the right from $f(x)$.
3 Determine the equations of the asymptotes using any translations from the basic curve $f(x) = \frac{1}{x}$.	As $g(x)$ is translated 3 units right, the vertical asymptote is $x = 3$.
4 Check the reasonableness of the solution.	A vertical asymptote occurs where the denominator is equal to 0: $x = 3$. A horizontal asymptote occurs where the fraction is equal to 0: $y = 0$.

5 Determine the coordinates of the y-intercept of $g(x)$.

For $x = 0$:

$g(0) = \frac{-2}{(0-3)} = \frac{2}{3}$

The coordinates of the y-intercept of $g(x)$ are $\left(0, \frac{2}{3}\right)$.

6 For hyperbolas $f(x) = \frac{a}{(x-b)}$, the x-axis is an asymptote, so no x-intercepts.

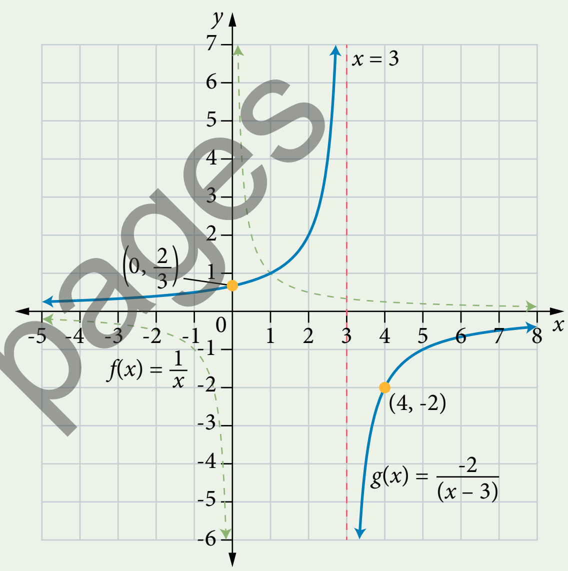
Determine the coordinates of a point on the second arm of the graph.

For $x = 4$:

$g(4) = \frac{-2}{(4-3)} = -2$

The point is at (4, -2).

7 Sketch the graphs of $f(x)$ and $g(x)$ showing the major features.



(c) $g(x) = \frac{-4}{x+2} + 1$

1 Write $g(x)$ in the form $f(x) = \frac{a}{(x-b)}$ and identify the values for a and b.

$g(x) = \frac{-4}{x+2} + 1$ $\quad a = -4$, $b = -2$ and $c = 1$

2 Identify and determine the significance of the a, b and c values relative to the graph of $f(x)$.

Compared to $y = \frac{1}{x}$:

For $|a| = 4$, the dilation factor is 4 parallel to the y-axis

$a < 0$ represents a reflection in the x-axis

$b = 2$ represents a horizontal translation 2 units left

$c = 1$ represents a vertical translation 1 unit up.

3 Determine the equations of the asymptotes using any translations from the basic curve $f(x) = \frac{1}{x}$.

As $g(x)$ is translated 2 units left, the equation of the vertical asymptote is $x = -2$.

As $g(x)$ is translated vertically 1 unit up, the equation of the horizontal asymptote is $y = 1$.

4 Check the reasonableness of the solution.

An asymptote occurs where the denominator is equal to 0: $x = -2$

An asymptote occurs where the fraction is equal to 0: $y = 1$

5 Determine the coordinates of the y-intercept of $g(x)$.

For $x = 0$:

$$g(0) = \frac{-4}{0+2} + 1 = -1$$

The coordinates of the y-intercept are (0, -1).

6 Determine the coordinates of the x-intercept of $g(x)$.

For $y = 0$:

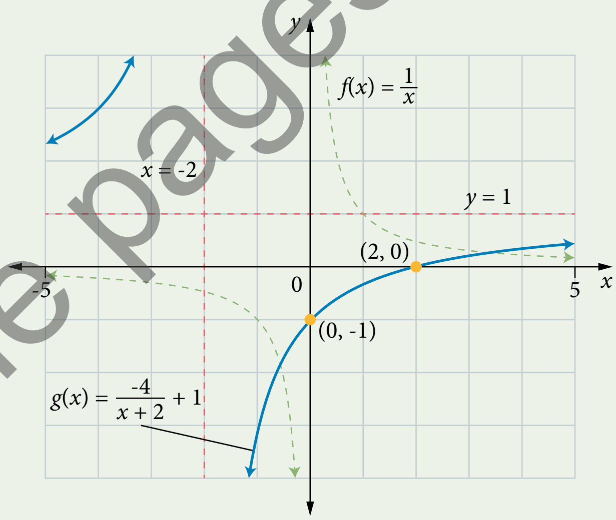
$$0 = \frac{-4}{x+2} + 1$$

$$4 = x + 2$$

$$x = 2$$

The coordinates of the x-intercept are (2, 0).

7 Sketch the graphs of $f(x)$ and $g(x)$, showing the major features.



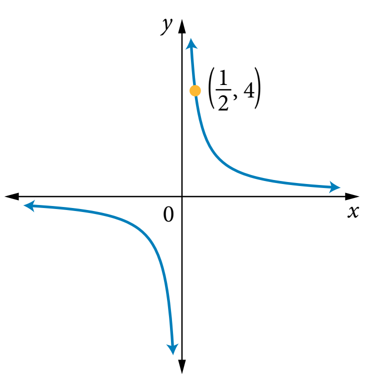
The rule for a rectangular hyperbola, when the asymptotes are known, can be determined from just one point.

Technology worked example
Graphing rectangular hyperbolas

5 Determine the rule for a rectangular hyperbola

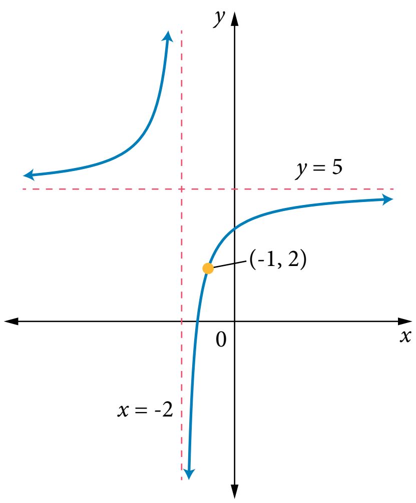
Determine the rule for each rectangular hyperbola.

(a)



THINKING	WORKING
1 The axes are asymptotes so use the general form for inverse proportion.	Let the equation be $y = \frac{k}{x}$.
2 Substitute the given point and solve for k.	For $\left(\frac{1}{2}, 4\right)$: $4 = \frac{k}{\frac{1}{2}}$ $k = 4 \times \frac{1}{2}$ $k = 2$
3 Write the equation.	$y = \frac{2}{x}$

(b)



1 When the asymptotes are not the axes, use the general form for rectangular hyperbola.	Let the equation be in the form $y = \frac{a}{(x-b)} + c$.
2 Substitute the parameters given by the asymptotes.	Vertical asymptote: $x = -2$, so $b = -2$ Horizontal asymptote: $y = 5$, so $c = 5$ The equation becomes $y = \frac{a}{x+2} + 5$
3 Substitute the given point and solve for a.	For $(-1, 2)$: $2 = \frac{a}{-1+2} + 5$ $2 = a + 5$ $a = -3$
4 Write the equation.	$y = \frac{-3}{x+2} + 5$
5 Check the reasonableness of your answer.	The graph shows a reflection in the x-axis compared to the graph of $y = \frac{1}{x}$, so a negative value of a was expected.

Algebraic manipulation to match the general form

The general equation of a rectangular hyperbola is $f(x) = \frac{a}{(x-b)} + c$, but it can also be represented by a linear function divided by another linear function. In this case, you can use equivalent fractions to obtain the general equation.

6 Algebraic manipulation to match the general form

Write $h(x) = \dfrac{5x+21}{x-2}$ in the form of the general equation for a rectangular hyperbola.

THINKING	WORKING
1 Equate the given expression to $\frac{a}{(x-b)}+c$, making the denominators equivalent. Use a common denominator with the RHS. Simplify.	$\dfrac{5x+21}{x-2} = \dfrac{a}{x-2} + c$ $= \dfrac{a}{x-2} + \dfrac{x(x-2)}{x-2}$ $= \dfrac{a+c(x-2)}{x-2}$ $= \dfrac{cx+a-2c}{x-2}$
2 Equate the coefficients of x in the numerator to solve for c.	$5x = cx$ $\therefore c = 5$
3 Equate the constant terms in the numerator and substitute for c to solve for a.	Given $c = 5$: $5x + 21 = cx + a - 2c$ $5x + 21 = 5x + a - 2 \times 5$ $a - 10 = 21$ $a = 31$
4 Write the function in the general form.	$h(x) = \dfrac{31}{x-2} + 5$

EXERCISE 4.1 Inverse proportion and graphs of rectangular hyperbolas

Worked Example

1 The time taken t (hours) to paint a mural on a large wall by n painters is given by the equation $t = \dfrac{36}{n}$. 1

(a) Determine the time it would take to paint the mural for each of the following numbers of painters.

(i) 3 (ii) 6 (iii) 9 (iv) 12

(b) Determine the number of painters required if the mural needs to be painted in 10 hours.

2 The relationship between the volume of an inflated balloon V (cm^3) with a surrounding air pressure P (kilopascals) is $V = \frac{200}{P}$.

(a) Calculate the volume of the balloon at sea level when the air pressure is 100 kilopascals.

(b) The balloon is inflated with helium and let go. As it flies higher, the surrounding air pressure decreases. Determine the volume of the balloon when the surrounding air pressure is:

(i) 50 kilopascals (ii) 25 kilopascals (iii) 1 kilopascal

Worked Example

3 The frequency of sound f (hertz) is related to the wavelength λ (m) by the relationship $f = \frac{300}{\lambda}$. Determine the frequency of sound with wavelength 0.2 m.

2

4 A time trial over the distance of 180 km involves five cars travelling at average speeds of 240 km/h, 180 km/h, 120 km/h, 90 km/h and 60 km/h.
The equation for the time of travel t (hours) for an average speed of s (km/h) is $t = \frac{180}{s}$.

(a) Complete the table of values for the time taken for the cars to complete the journey.

Average speed (km/h)	60	90	120	180	240
Time (h)					

(b) Sketch the graph and explain the shape of the curve.

3

5 Soluble vitamin tablets often dissolve quicker when the water is warmer. Sian dissolved some vitamin tablets in water at different temperatures, T°C, and measured the time t (s) it took for one tablet to dissolve in a fixed quantity of water.

T°C	t (s)
8	120
15	64
20	48

Determine the rule of the function connecting T°C and t (s).

6 y and x are inversely proportional, and the point (3.5, 41) is on this graph.

(a) Calculate y where $x = 17.5$.

(b) Calculate x where $y = 51.25$.

7 y and x are inversely proportional, and the point (0.8, 0.075) is on this graph.

(a) Calculate y (to 1 decimal place) where $x = 0.2$.

(b) Calculate x (to 4 decimal places) where $y = 37.5$.

8 For the function $y = -\frac{2}{x+1} + 1$, determine the y-value for $x = 1$.

9 As a piston is pushed down into a cylinder, the volume decreases and the pressure increases. The pressure P (Pa) inside the cylinder is inversely proportional to the volume V (cm^3). If the volume in the piston is 9 cm^3 when the pressure is 0.3 Pa, determine the pressure when the volume is 20 cm^3.

10 The resistance of an electrical appliance is inversely proportional to the current applied under constant voltage. The equation for the resistance R (ohm) when a current I (amp) is applied through an appliance is $R = \frac{360}{I}$.

(a) Complete the table of values for the resistance when the current is applied through the appliance.

(b) Draw a graph of resistance versus current.

(c) Explain the shape of the curve.

Current, I (amps)	10	30	60	120	360
Resistance, R (ohms)					

11 Determine the equations of the asymptotes for $f(x) = \dfrac{2}{x+3} - 4$.

A $x = 3, y = 2$ **B** $x = -3, y = 2$ **C** $x = 3, y = 4$ **D** $x = -3, y = -4$

12 Consider the function $g(x) = \dfrac{-4}{x-1} + 2$.

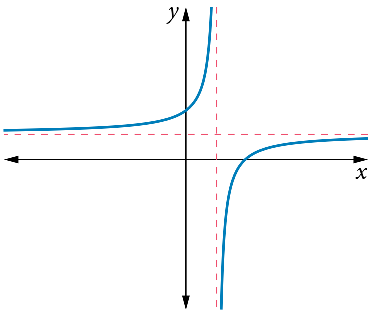
(a) Determine the coordinates of the x-intercept and y-intercept respectively.

A (3, 0), (0, 6) **B** (-3, 0), (0, 6) **C** (0, 3), (6, 0) **D** (4, 0), (0, 2)

(b) Explain the common error made by a student who thought the answer was $(1, 0), (0, 2)$.

13 Choose the graph that represents $f(x) = -\dfrac{1}{x-1} + 1$.

A



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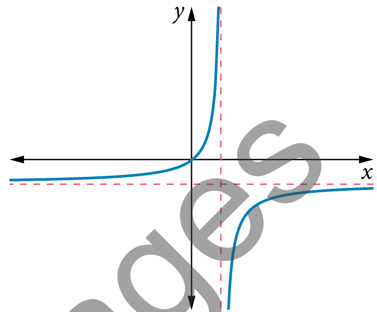
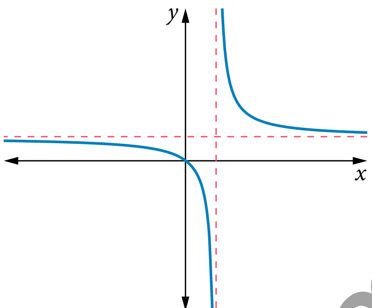
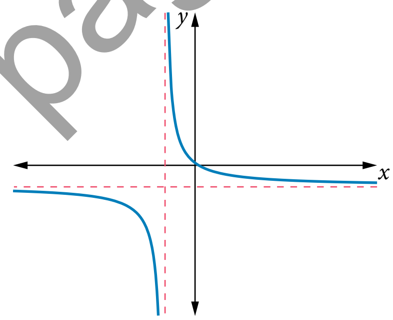


C



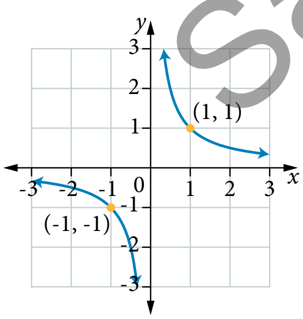
D



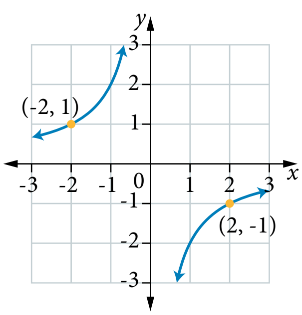
Worked Example 5

14 Determine the equation for each of the following hyperbolic graphs.

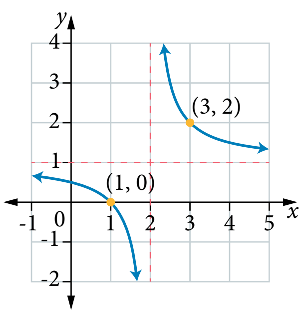
(a)



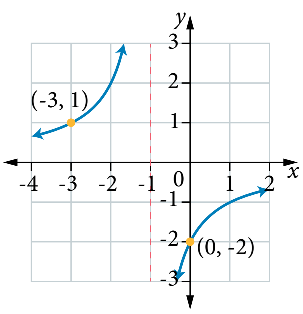
(b)



(c)



(d)



Worked Example

6 **15** Express each of the following in the form $\frac{a}{x-b}+c$.

(a) $\frac{x+4}{x-5}$ **(b)** $\frac{x-1}{x+2}$ **(c)** $\frac{x}{x+3}$ **(d)** $\frac{2x+3}{x+1}$ **(e)** $\frac{5-x}{x+4}$

4 **16** For each of the following functions, complete the table of values, write the equations of the asymptotes, sketch the graph of the function and label one point on the graph that belongs to it.

(a) $y=\frac{1}{x-4}$

x	1	2	3	4	5	6	7
y							

(b) $y=\frac{1}{x+6}$

x	-9	-8	-7	-6	-5	-4	-3
y							

17 For each of the following functions, complete the table of values and determine the equations of the asymptotes, sketch the graph of the function and label one point on the graph.

(a) $y=\frac{2}{x}$

x	-2	-1	$-\frac{1}{2}$	$\frac{1}{2}$	1	2
y						

(b) $y=\frac{1}{2x}$

x	-2	-1	$-\frac{1}{2}$	$\frac{1}{2}$	1	2
y						

(c) $y=\frac{1}{x}+4$

x	-3	-2	-1	0	1	2	3
y							

(d) $y=\frac{1}{x}-6$

x	-3	-2	-1	0	1	2	3
y							

18 Sketch the graph of each function, showing the coordinates of intercepts and equations of asymptotes.

(a) $y=\frac{1}{x-2}+4$ **(b)** $y=\frac{4}{x-1}-6$ **(c)** $y=2+\frac{2}{x+1}$ **(d)** $y=-3+\frac{3}{x+6}$

(e) $y=\frac{-2}{x+1}-1$ **(f)** $y=6-\frac{1}{x+4}$ **(g)** $y=-\frac{1}{x+4}-2$ **(h)** $y=2+\frac{1}{4-x}$

(i) $f(x)=\frac{3x+7}{x+2}$ **(j)** $f(x)=\frac{4x+11}{x-3}$ **(k)** $f(x)=\frac{3x-7}{x+2}$ **(l)** $f(x)=\frac{4x-11}{x+3}$

19 Sketch the graph of $y=\frac{2x+5}{3-4x}$, by first determining the equations to the asymptotes, and the coordinates of the x- and y-intercepts.

20 $f(x)=g(x)-h(x)$ where $g(x)=\frac{2}{x-1}+3$ and $h(x)=\frac{4}{3-x}$.

Determine the subdomains where $f(x)>0$.

The graph of $x = y^2$

4.2

The inverse of $y = x^2$

An inverse relation can be produced by reversing the independent and dependent variables. The relation $x = y^2$ is not a function because the equation can be written as $y = \pm\sqrt{x}$, $x \geq 0$, and hence there are two y-values for each x-value in the domain (except for zero).

Interchanging the x- and y-coordinates of every point in the Cartesian plane is a transformation of the plane that maps the graph of a relation onto the graph of its inverse relation, and vice versa. Geometrically, this corresponds to a reflection in the line $y = x$ as long as the axes have one-to-one scales.

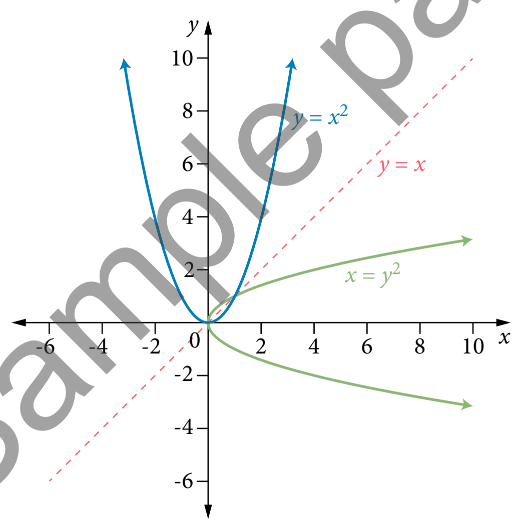
The following table of values represents part of the relation $y = x^2$.

x	-3	-2	-1	0	1	2	3
y	9	4	1	0	1	4	9

The corresponding values for the inverse relation $x = y^2$ or $y = \pm\sqrt{x}$ are shown in the table on the right.

x	0	1	4	9
y	0	±1	±2	±3

The following shows the graphs of both relations on the same set of axes, using a one-to-one ratio for the axes scales.



The graph of $y = x^2$ intersects with the graph of $x = y^2$ on the line $y = x$ at (0, 0) and (1, 1).

The function $y = \sqrt{x}$

The domain of the function $f(x) = \sqrt{x}$ is the set of non-negative numbers $[0, \infty)$ or $x \geq 0$.

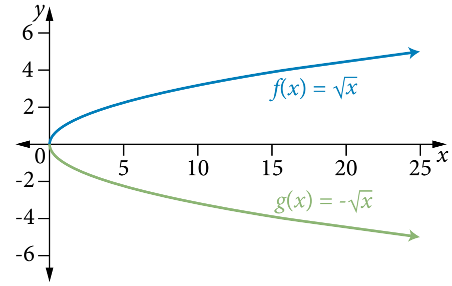
The following table shows some values for $f(x) = \sqrt{x}$.

x	0	1	4	9	16	25
y	0	1	2	3	4	5

The function $f(x) = \sqrt{x}$ corresponds to the positive (and zero) part of the relation $x = y^2$. The range is also the set of non-negative numbers $[0, \infty)$ or $y \geq 0$.

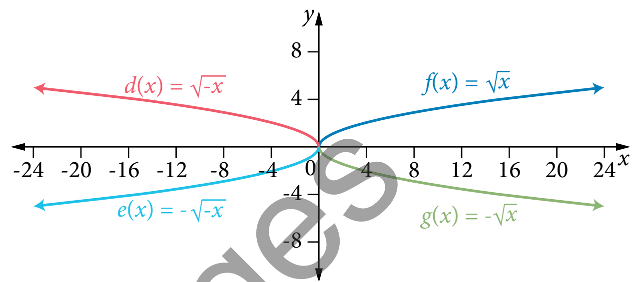
The function $g(x) = -\sqrt{x}$ corresponds to the negative (and zero) part of the relation $x = y^2$.

The graph of $g(x) = -\sqrt{x}$ is the reflection of the graph of $f(x) = \sqrt{x}$ in the x-axis.



The graph of the function $d(x) = \sqrt{-x}$ is the reflection of the graph of $f(x)$ in the y-axis.

The graph of the function $e(x) = -\sqrt{-x}$ is the reflection of the graph of $g(x)$ in the y-axis.



You can graph a relation such as $x = y^2$ by drawing both square root functions: the positive arm and the negative arm. The graphs of these relations are parabolas so have a horizontal axis of symmetry and a vertex.

The vertex of $x = y^2$ is (0, 0).

The line of symmetry is the x-axis: $y = 0$.

WARNING

The vertex of $x = y^2$ is not a turning point as the gradient of the tangent at the vertex is undefined.

7 Graph relations by forming two functions

Sketch the graph of the following relations.

(a) $x = y^2 + 2$

THINKING	WORKING
1 Use your understanding of the inverse relation and the graph of $y = x^2$ to predict the shape of the graph.	For $y = x^2 + 2$, the vertex is at (0, 2). The line of symmetry is the y-axis: $x = 0$. For $x = y^2 + 2$: the vertex is at (2, 0). The line of symmetry is the x-axis: $y = 0$.
2 Solve the equation to make y the subject and identify the domain.	$x = y^2 + 2$ $y^2 = x - 2$ $y = \pm\sqrt{x-2}$, $x \geq 2$

3 Determine the coordinates of an additional point on each arm. Use your knowledge of perfect squares.

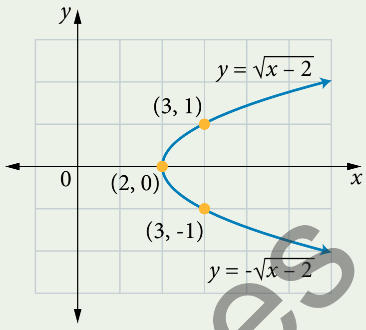
For $x = 3$:

$$\begin{aligned} y &= \pm\sqrt{3-2} \\ &= \pm\sqrt{1} \\ &= \pm 1 \end{aligned}$$

$y = \sqrt{x-2}$ passes through (3, 1).

$y = -\sqrt{x-2}$ passes through (3, -1).

4 Sketch the graph. Confirm the shape of the graph using technology.



(b) $x = (y+2)^2$

1 Use your understanding of the inverse relation and the graph of $y = x^2$ to predict the shape of the graph.

For $y = (x+2)^2$, the vertex is at (-2, 0). The line of symmetry is the line of $x = -2$.

For $x = (y+2)^2$, the vertex is at (0, -2). The line of symmetry is the line $y = -2$.

2 Solve the equation to make y the subject and identify the domain.

$$\begin{aligned} x &= (y+2)^2 \\ y + 2 &= \pm\sqrt{x} \\ y &= -2 \pm \sqrt{x},\ x \geq 0 \end{aligned}$$

3 Determine the coordinates of an additional point on each arm. Use your knowledge of perfect squares.

For $x = 4$:

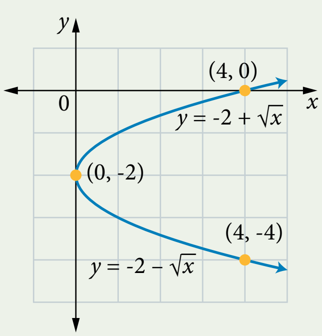
$$\begin{aligned} y &= -2 \pm \sqrt{4} \\ &= -2 \pm 2 \\ &= -4 \text{ or } 0 \end{aligned}$$

$y = -2 + \sqrt{x}$ passes through (4, 0).

$y = -2 - \sqrt{x}$ passes through (4, -4).

4 Sketch the graph. Confirm the shape of the graph using technology.

$y = -2 + \sqrt{x}$ and $y = -2 - \sqrt{x}$



(c) $x = -y^2$

1 Use your understanding of the inverse relation and the graph of $y = x^2$ to predict the shape of the graph.

For $y = -x^2$, the vertex is (0, 0).
The line of symmetry is the y-axis, $x = 0$.

For $x = -y^2$, the vertex is at (0, 0).
The line of symmetry is the x-axis, $y = 0$.

2 Solve the equation to make y the subject.

$$x = -y^2$$
$$y^2 = -x$$
$$y = \pm\sqrt{-x},\ x \le 0$$

3 Determine the coordinates of an additional point on each arm. Use your knowledge of perfect squares.

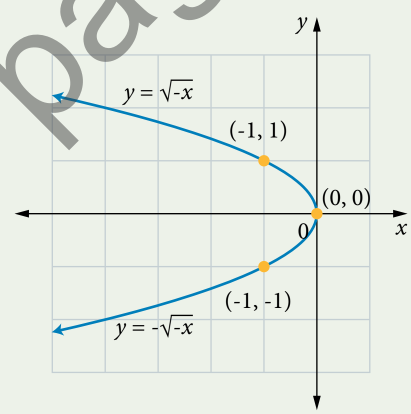
For $x = -1$

$$y = \pm\sqrt{-(-1)}$$
$$= \pm\sqrt{1}$$
$$= \pm 1$$

$y = \sqrt{-x}$ passes through (-1, 1).
$y = -\sqrt{-x}$ passes through (-1, -1).

4 Sketch the graph. Confirm the shape of the graph using technology.

$y = \sqrt{-x}$ and $y = -\sqrt{-x}$



Square root functions

Transformations of the basic square root function have many practical applications such as calculating the period of swing for a pendulum from its length, calculating the speed of a car from the length of the skid marks after an accident and determining the speed of sound from the air temperature.

You can create square root functions by changing the subject of a formula if one of the variables is squared. In practical situations where neither variable can be negative, this generally leads to a single function because you reject the negative expression.

8 Calculate a length, given the area

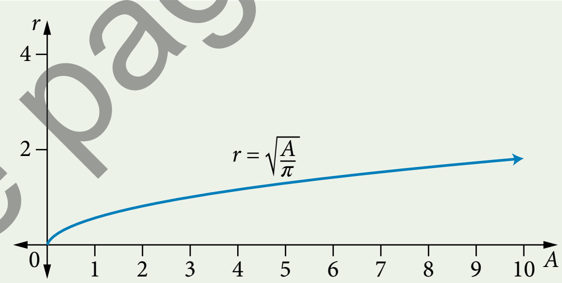
Consider the formula for area of a circle $A = \pi r^2$.

(a) Construct a formula for radius in terms of area.

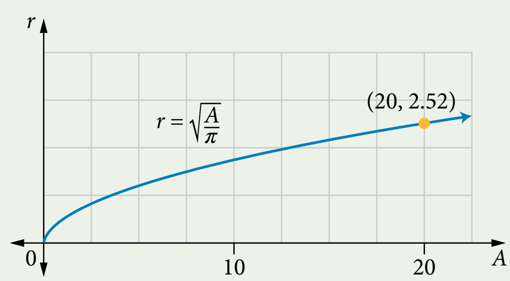
THINKING	WORKING
1 Transpose the equation to make the radius the subject.	$A = \pi r^2$ $r^2 = \frac{A}{\pi}$ $r = \pm\sqrt{\frac{A}{\pi}}$
2 Reject the negative expression, because lengths cannot be negative.	The radius r represents a length $\therefore r \geq 0$ $r = \sqrt{\frac{A}{\pi}}$

(b) Graph the function using technology.

Use technology to draw $y = \sqrt{\frac{x}{\pi}}$. Replace the variables as appropriate.



(c) Determine the radius, given that the area is $20\,\text{cm}^2$.

1 Solve algebraically by substitution.	For $A = 20$: $r = \sqrt{\frac{A}{\pi}}$ $= \sqrt{\frac{20}{\pi}}$ $= 2.52\ (2\text{ d.p.})$
2 Interpret the result, including units.	A circle of area $20\,\text{cm}^2$ has a radius of 2.52 cm.
3 Check the reasonableness of the coordinates by locating the point on your graph.	

Transformations of the square root function

The basic square root graph of $y = \sqrt{x}$ can be translated, dilated or reflected to give some recognisable general forms.

The general form for transformations of the function $y = \sqrt{x}$ is $f(x) = a\sqrt{(x-b)} + c$, where:

- $|a|$ is the vertical dilation factor parallel to the y-axis. When a is positive, the graph is the positive arm of the parabola and when a is negative, the graph is the negative arm of the parabola
- b translates the function horizontally, parallel to the x-axis
- c translates the function vertically, parallel to the y-axis.

The point (b, c) is the vertex of the graph.

The function $y = \sqrt{-x}$ is the reflection in the y-axis of $y = \sqrt{x}$.

The general form for transformations of the function $y = \sqrt{-x}$ is $f(x) = a\sqrt{-(x-b)} + c$, where:

- $|a|$ is the vertical dilation factor parallel to the y-axis. When a is positive, the graph is the positive arm of the parabola and when a is negative, the graph is the negative arm of the parabola
- b translates the function horizontally, parallel to the x-axis
- c translates the function vertically, parallel to the y-axis.

The point (b, c) is the vertex of the graph.

Making connections

Transforming the square root graph

Move the sliders to view the effect each parameter has on the graph of $y = a\sqrt{(x-b)} + c$.

Technology worked example

Graphing the square root function

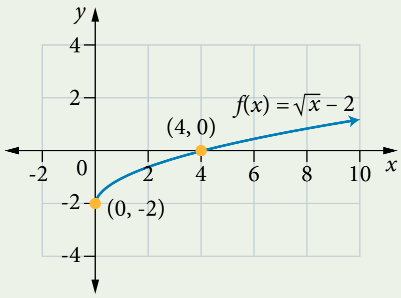
9 Sketch simple square root functions

Sketch the graph of $f(x)$, clearly showing the coordinates of the vertex and axes intercepts. Identify the domain and range.

(a) $f(x) = \sqrt{x} - 2$

THINKING	WORKING
1 Identify the transformations that have occurred from the standard square root function $f(x) = \sqrt{x}$.	From the graph of $y = \sqrt{x}$, $f(x) = \sqrt{x} - 2$ has been translated 2 units down.
2 Determine the coordinates of the vertex.	The coordinates of the vertex are (0, -2).
3 Determine the coordinates of the axes intercept of $f(x)$.	The y-intercept is the vertex. $\sqrt{x} - 2 = 0$ $\sqrt{x} = 2.$ $x = 4$ The coordinates of the x-intercept are (4, 0).

4 Sketch the graph of $f(x)$ showing the vertex and x-intercept.



5 Identify the domain and range of the function using interval and set notation.

The domain of $f(x)$ is $x \geq 0$ or $[0, \infty)$.

The range of $f(x)$ is $y \geq -2$ or $[-2, \infty)$.

(b) $f(x) = \sqrt{-x + 2}$

1 Rewrite the rule so transformations can be identified and state the domain.

$$f(x) = \sqrt{-x+2}$$
$$= \sqrt{-(x-2)},\ x \leq 2$$

2 Identify the transformations that have occurred from the function $f(x) = \sqrt{-x}$.

From the graph of $f(x) = \sqrt{-x}$, $f(x) = \sqrt{-x+2}$ has been translated 2 units right.

3 Determine the coordinates of the vertex.

The coordinates of the vertex are (2, 0).

4 Determine the coordinates of the axes intercept of $f(x)$.

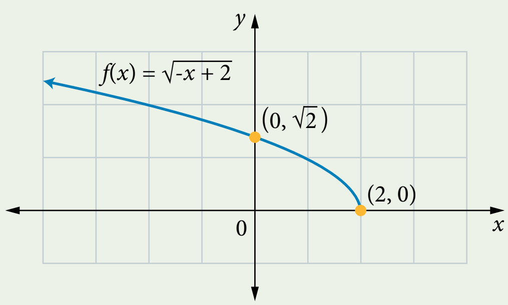
The x-intercept is the vertex.

For $x = 0$:

$$f(x) = \sqrt{-x+2}$$
$$f(0) = \sqrt{-0+2}$$
$$= \sqrt{2}$$

The coordinates of the y-intercept are $\left(0, \sqrt{2}\right)$.

5 Sketch the graph of $f(x)$ showing the vertex and x-intercept.



6 Identify the domain and range of the function using interval and set notation.

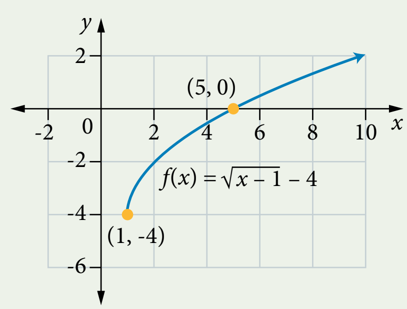
The domain of $f(x)$ is $x \leq 2$ or $(-\infty, 2]$.

The range of $f(x)$ is $y \geq 0$ or $[0, \infty)$.

10 Sketch square root functions with rule in the form $f(x) = a\sqrt{(x-b)} + c$

Consider the function $f(x) = 2\sqrt{x-1} - 4$.

(a) Sketch the graph of $f(x)$, clearly showing the coordinates of the vertex and axis intercept.

THINKING	WORKING
1 Write the rule in the form $f(x) = a\sqrt{(x-b)} + c$ and identify the domain.	$f(x) = 2\sqrt{(x-1)} - 4,\ x \geq 1$
2 Identify the values for a, b and c.	$a = 2$, $b = 1$ and $c = -4$
3 State the significance of the a, b and c values relative to the graph of $y = \sqrt{x}$.	For $a = 2$, the dilation factor is 2 parallel to the y-axis showing the upper arm of the parabola. For $b = 1$, the graph of $f(x)$ is translated 1 unit to the right from the origin. For $c = -4$, the graph of $f(x)$ is translated 4 units down.
4 Determine the coordinates of the vertex.	Considering the transformations stated above the coordinates of the vertex are (1, -4).
5 Determine the coordinates of the axes intercepts.	From the position of the vertex at (1, -4), the graph does not cut the y-axis. For $f(x) = 0$: $2\sqrt{x-1} - 4 = 0$ $2\sqrt{x-1} = 4$ $\sqrt{x-1} = 2$ $x - 1 = 4$ $x = 5$ The coordinates of the x-intercept are (5, 0).
6 Sketch the graph of $f(x)$ showing the vertex and x-intercept.	

(b) Identify the domain and range of the function.

Identify the domain and range of the function using interval and set notation.	The domain of $f(x)$ is $x \geq 1$ or $[1, \infty)$. The range of $f(x)$ is $y \geq -4$ or $[-4, \infty)$.

EXERCISE 4.2 The graph of $x = y^2$

Worked Example

1 Using technology or otherwise, sketch the graph of each relation. Identify the coordinates of the vertex and intercepts. 7

(a) $x = 3y^2$ (b) $y^2 = 3x$ (c) $x = y^2 + 3$ (d) $y^2 = x + 3$

2 Consider the relation $x = y^2 + 1$.

(a) Complete the table of values and hence sketch the graph of the relation.

(b) Determine the coordinates of the vertex and the equation of the line of symmetry.

x							
y	-3	-2	-1	0	1	2	3

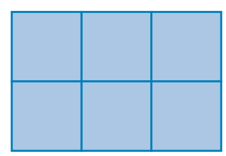
3 Consider the relation $x = (y+1)^2$.

(a) Complete the table of values and hence sketch the graph of the relation.

x							
y	-4	-3	-2	-1	0	1	2

(b) Determine the coordinates of the vertex and the equation of the line of symmetry.

4 The shape given in the diagram is made up of six squares of side length x cm. 8



(a) Construct a formula for x in terms of the area of the shape $A\,\text{cm}^2$.

(b) Sketch a graph of x as a function of A.

(c) Determine the value of x, correct to 2 decimal places, when the area of the shape is $20\,\text{cm}^2$. Confirm the answer from your graph.

5 For a particular type of material, the formula for the diameter D (cm) of rope needed to lift a load of mass m (tonne) is $D = 0.5\sqrt{m}$.

(a) Determine the diameter needed for a mass of 1.5 tonnes. Give your answer correct to 2 decimal places.

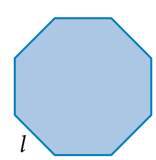
(b) Determine the mass that could be lifted by a rope of diameter 1.5 cm. Give your answer correct to 2 decimal places.

6 An approximate formula for calculating distance to the horizon D (km) in terms of altitude A (m) is $D = 3.57\sqrt{A}$.

(a) Determine the distance to the horizon from an aircraft flying at 10 000 m.

(b) How would the distance to the horizon change if the aircraft dropped down by 3000 m? Give your answer to the nearest kilometre.

7 The side length of a regular octagon in terms of its area is given by $l = \sqrt{\dfrac{A}{4.828}}$.



(a) Determine the side length of an octagon of area $200\,\text{cm}^2$. Give your answer correct to 2 decimal places.

(b) Determine the area of a stop sign of side length 28.7 cm. Give your answer correct to the nearest square centimetre.

8 Consider the relation $x = y^2 + 4$.

(a) Determine the value for x when $y = -2$.

A 8 B 2 C -8 D -2

(b) Explain the common error made by a student who thought the answer was 0.

9 For $y = 2\sqrt{x+1} - 4$, determine the y-value for $x = 3$.

10 For $y = \sqrt{x-2}$, determine the x-value for $y = 2$.

11 For $y = -2\sqrt{x+9} + 4$, the coordinates of the vertex are:

A (-2, -9) B (-9, 4) C (9, 4) D (-2, 4)

Worked Example 9

12 Determine the coordinates of the y-intercept for $y = -\sqrt{2x+4} - 5$.

13 For each of the following functions sketch the graph showing the coordinates of the vertex and any axes intercepts. Include an additional point, if necessary, to show the scale. Identify the domain and range using interval notation.

(a) $f(x) = \sqrt{x} + 2$ (b) $f(x) = \sqrt{x} - 4$ (c) $f(x) = 2\sqrt{x} + 3$

(d) $y = \sqrt{x+2}$ (e) $y = \sqrt{-x-1}$

14 For each of the following functions, sketch the graph showing the vertex and any axis intercepts. Identify the domain and range.

(a) $f(x) = -\sqrt{x}$ (b) $f(x) = -\sqrt{x} + 4$

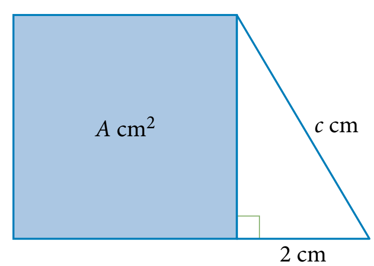
(c) $f(x) = -\sqrt{x} - 1$ (d) $f(x) = -2\sqrt{x} - 4$

15 Sketch the following functions using technology and identify the coordinates of the vertex and axis intercepts.

(a) $f(x) = 3\sqrt{x-1} + 2$ (b) $f(x) = \sqrt{x+9} - 1$

(c) $f(x) = \sqrt{9-x} - 4$ (d) $f(x) = -\sqrt{4-x} + 4$

16 A right-angled triangle has a side length of 2 cm and hypotenuse of c cm. A square of area A cm^2 is drawn on the third side. Create a formula for c as a function of A. Graph the function for areas up to 60 cm^2.



17 Skid marks are often left on the road after a road accident. For a particular car in certain conditions the formula for the speed s (km/h) prior to braking in terms of the length of the skid marks l (m) is given as $s = \sqrt{185l}$.

(a) Determine whether a car leaving skid marks of 12.5 m was travelling above the speed limit in a 60 km/h zone.

(b) A stunt driver travelling at 80 km/h on a similar surface locks the brakes so that the car skids towards a wall 35 m away. Determine how close the car comes to the wall.

18 For each of the following functions, sketch the graph showing the coordinates of the vertex and any axis intercepts. Identify the domain and range.

(a) $f(x) = \sqrt{x+1} - 2$
(b) $f(x) = \sqrt{x-4} + 4$
(c) $f(x) = -\sqrt{x+1} + 3$
(d) $f(x) = -2\sqrt{x-9} - 2$

19 For each of the following functions, sketch the graph showing the vertex and any intercepts. Identify the domain and range.

(a) $f(x) = \sqrt{-(x+1)} - 2$
(b) $f(x) = \sqrt{-(x-4)} + 3$
(c) $f(x) = -\sqrt{-(x+1)} - 2$
(d) $f(x) = -2\sqrt{-(x-4)} + 8$

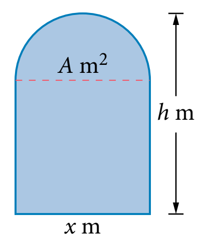
20 Consider the function $f(x) = -2\sqrt{-2\left(x - \frac{5}{2}\right)} + 8$.

(a) Sketch the graph of the function showing the coordinates of each major point (vertex and intercepts as relevant).
(b) Determine the domain and range of $f(x)$.

21 Consider the function $f(x) = 3\sqrt{\left(\frac{1}{2} + 2x\right)} - 1$.

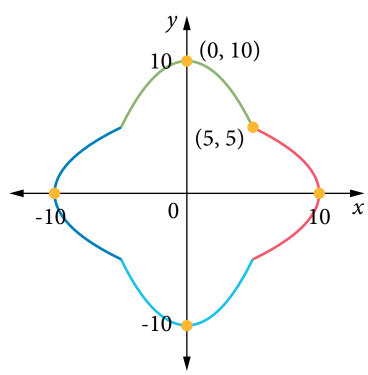
(a) Sketch the graph of the function showing the coordinates of each major point (vertex and intercepts as relevant).
(b) Determine the domain and range of the function.

22 The shape on the right consists of a semicircle on a square of side x m.



(a) Create a formula for x in terms of the area A (m^2) in the form $x = k\sqrt{A}$ and graph this function. Label the point where $A = 45$ on the graph.
(b) The height of the shape is h (m). Create a formula for h in terms of the area.
(c) Describe the transformation that has occurred to produce the formula for h from the formula for x.

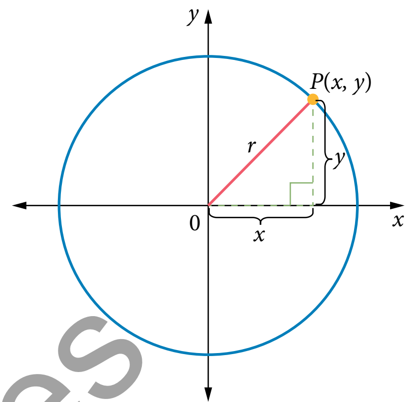
23 The symmetrical diagram below consists of four congruent parabolas. Create an equation for each parabola, and then make two piece-wise functions to replicate the shape.



4.3 Circles

Circle with centre at the origin

Consider the point $P(x, y)$ on a circle with its centre at the origin. A vertical line segment drawn from P to the x-axis forms a right-angled triangle. Let the hypotenuse of this triangle be of length r.



From Pythagoras' theorem, the circle with centre at the origin and radius r is the set of ordered pairs (x, y) that satisfies the relation $x^2 + y^2 = r^2$.

The general equation for any circle with its centre at the origin and radius r is $x^2 + y^2 = r^2$.

WARNING

Draw circles with the same scale on both axes. If you don't, the shape will be distorted like an oval and the value of the radius will not be meaningful.

Additional information

The circle formula

Investigate how the equation of a circle is derived.

11 Sketch graphs of circles centred at the origin

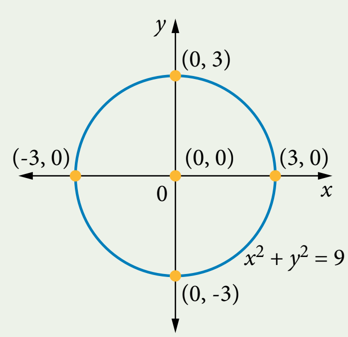
Consider the relation $x^2 + y^2 = 9$.

(a) Identify the centre and radius of the circle.

THINKING	WORKING
1 Write the rule in the form $x^2 + y^2 = r^2$ to identify the value of r.	$x^2 + y^2 = 9$ $x^2 + y^2 = 3^2$
2 Determine the centre and radius.	The centre is at the origin (0, 0). For $r = 3$, the length of the radius of the circle is 3 units.

(b) Sketch the graph of the relation.

Sketch the graph of the relation showing the centre and x- and y-axis intercepts.

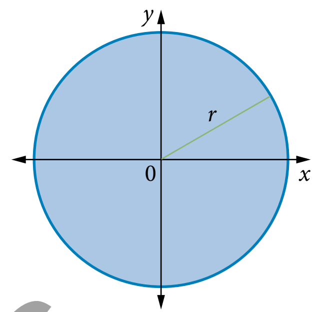


(c) Identify the domain and range of the relation.

Write the domain and range of the relation using interval notation.	The domain of the relation is [-3, 3]. The range of the relation is [-3, 3].

Points within the boundary

Any point lying within or on a circle with centre at the origin and radius r satisfies the inequality $x^2 + y^2 \le r^2$.



Additional information

Circles with a centre at the origin

Explore writing the equation of circles with centre at the origin.

12 Points lying within the boundary of a circle

Determine whether each point lies inside, outside or on the circle $x^2 + y^2 = 25$.

(a) (4, -3)

THINKING	WORKING
1 Calculate the value of $x^2 + y^2$.	For (4, -3): $x^2 + y^2 = 4^2 + (-3)^2$ $= 16 + 9$ $= 25$
2 Compare $x^2 + y^2$ to r^2.	$r^2 = 25$ $x^2 + y^2 = r^2$ $25 = 25$
3 Interpret the result.	$x^2 + y^2 = r^2$, so the point is on the circle.

(b) (-4.5, 2.5)

1 Calculate the value of $x^2 + y^2$.	For (-4.5, 2.5): $x^2 + y^2 = (-4.5)^2 + 2.5^2$ $= 20.25 + 6.25$ $= 26.5$
2 Compare $x^2 + y^2$ to r^2.	$r^2 = 25$ $x^2 + y^2 > r^2$ $26.5 > 25$
3 Interpret the result.	$x^2 + y^2 > r^2$, so the point is outside the circle.

(c) (-3.5, -1.2)

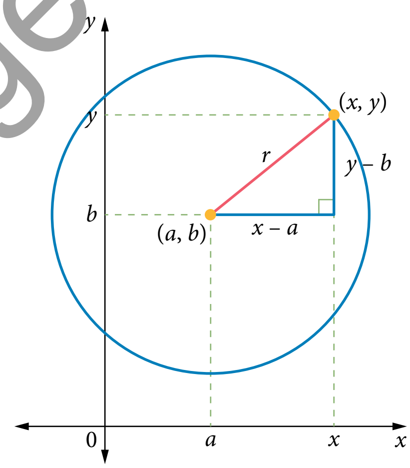
1 Calculate the value of $x^2 + y^2$.	For (-3.5, -1.2): $x^2 + y^2 = (-3.5)^2 + (-1.2)^2$ $= 12.25 + 1.44$ $= 13.69$
2 Compare $x^2 + y^2$ to r^2.	$r^2 = 25$ $x^2 + y^2 < r^2$ $13.69 < 25$
3 Interpret the result.	$x^2 + y^2 < r^2$, so the point is inside the circle.

Transformations and the graph of the circle with equation $(x - a)^2 + (y - b)^2 = r^2$

Technology worked example

Graphing circles

If a translation of a units parallel to the x-axis and b units parallel to the y-axis is applied to the relation $x^2 + y^2 = r^2$, the general form of a circle with centre (a, b) and radius r is obtained from Pythagoras' theorem, where $(x - a)^2 + (y - b)^2 = r^2$.



The general form of a circle of radius r is $(x - a)^2 + (y - b)^2 = r^2$, where the relation $x^2 + y^2 = r^2$ has undergone:

- a horizontal translation of a units parallel to the x-axis. If $a > 0$, the translation is right. If $a < 0$, the translation is left
- a vertical translation of b units parallel to the y-axis. If $b > 0$, the translation is up. If $b < 0$, the translation is down.

The centre of the circle is at (a, b).

WARNING

The centre of a circle is often displayed on the graph, but it is not a point belonging to the graph of the circle.

Making connections

Transforming a circle

Move the sliders to view the effect each parameter has on the relation $(x - a)^2 + (y - b)^2 = r^2$.

13 Sketch circles with centre (a,b)

Consider the relation with equation $(x - 2)^2 + (y + 1)^2 = 4$.

(a) Determine the centre and radius of the circle.

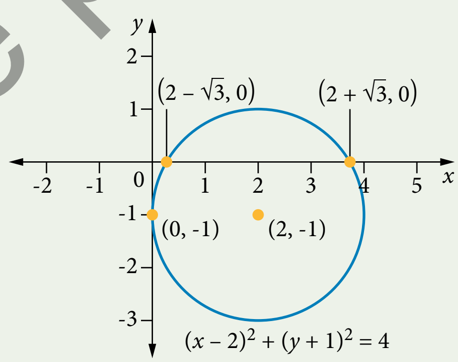
THINKING	WORKING
1 Write the equation in the form $(x - a)^2 + (y - b)^2 = r^2$.	$(x - 2)^2 + (y + 1)^2 = 4$ $(x - 2)^2 + (y + 1)^2 = 2^2$
2 Determine the coordinates of the centre and the length of the radius.	The coordinates of the centre are (2, -1) and the length of the radius is 2 units.

(b) Determine the axes intercepts.

1 Determine the coordinates of the x-intercepts.	x-intercept, let $y = 0$: $$(x-2)^2 + (1)^2 = 4$$ $$(x-2)^2 = 3$$ $$x - 2 = \pm\sqrt{3}$$ $$x = 2 \pm \sqrt{3}$$ The coordinates of the x-intercepts are $\left(2-\sqrt{3}, 0\right)$ and $\left(2+\sqrt{3}, 0\right)$.
2 Determine the coordinates of the y-intercept(s).	y-intercept, let $x = 0$: $$(-2)^2 + (y+1)^2 = 4$$ $$4 + (y+1)^2 = 4$$ $$(y+1)^2 = 0$$ $$y + 1 = 0$$ $$y = -1$$ The coordinates of the y-intercept are (0, -1).

(c) Sketch the curve of the relation.

Sketch the graph of the relation showing the centre and axes intercepts.



(d) Identify the domain and range of the relation.

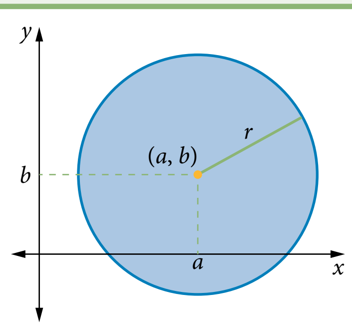
Write the domain and range of the relation using either interval notation or inequalities.	The domain is [0, 4] or $0 \le x \le 4$. The range is [-3, 1] or $-3 \le y \le 1$.

Additional information

The equation of a circle

Consolidate your understanding of the equation of the circle.

Any point lying within or on the a circle with centre (a, b) and radius r satisfies the inequality $(x-a)^2 + (y-b)^2 \le r^2$.



14 Points lying within the boundary of a circle with centre (a, b)

Determine whether each point lies inside, outside or on the circle $(x+4)^2 + (y-2)^2 = 9$.

(a) (3, 3)

THINKING	WORKING
1 Calculate the value of $(x-a)^2 + (y-b)^2$.	For (3, 3): $(x+4)^2 + (y-2)^2 = (3+4)^2 + (3-2)^2$ $= 7^2 + 1^2$ $= 50$
2 Identify the value of r^2.	$r^2 = 9$
3 Interpret the result.	$50 > 9$ $(x-a)^2 + (y-b)^2 > r^2$ The point is outside of the circle.

(b) (-3, 4.5)

THINKING	WORKING
1 Calculate the value of $(x-a)^2 + (y-b)^2$.	For (-3, 4.5): $(x+4)^2 + (y-2)^2 = (-3+4)^2 + (4.5-2)^2$ $= 1^2 + 2.5^2$ $= 7.25$
2 Identify the value of r^2.	$r^2 = 9$
3 Interpret the result.	$7.25 < 9$ $(x-a)^2 + (y-b)^2 < r^2$ The point is inside the circle.

Completing the square

Relations containing the terms x^2 and y^2 (or the same multiples of these, such as $3x^2$ and $3y^2$) and terms x and y, as well as a constant term could be formed into the general equation for a circle $(x-a)^2 + (y-b)^2 = r^2$ by completing the square. If this cannot be done, then there will be no points that satisfy the relation.

15 Use algebra to determine the centre and radius of a circle

By writing the equation $x^2 + 4x + y^2 - 6y = 12$ in the form $(x-a)^2 + (y-b)^2 = r^2$, demonstrate that the relationship is a circle and determine its centre and radius.

THINKING	WORKING
1 Use the process of completing the square on both the group of x-values and the group of y-values. Simplify by writing the constant terms on the RHS.	$x^2 + 4x + y^2 - 6y = 12$ $(x^2 + 4x + 4) - 4 + (y^2 - 6y + 9) - 9 = 12$
2 Simplify the equation by collecting like terms and writing in the form $(x-a)^2 + (y-b)^2 = r^2$.	$(x^2 + 4x + 4) + (y^2 - 6y + 9) - 13 = 12$ $(x+2)^2 + (y-3)^2 = 25$ $(x+2)^2 + (y-3)^2 = 5^2$
3 Determine the coordinates of the centre and the length of the radius.	The coordinates of the centre are (-2, 3) and the length of the radius is 5 units.

Semicircles

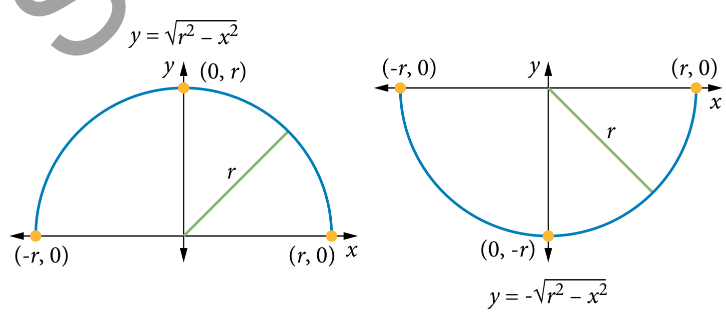
Semicircles centred at the origin

A semicircle is half a circle.

Transposing the general equation of a circle, centred at the origin with radius r, to make y the subject gives:

$$x^2 + y^2 = r^2$$
$$y^2 = r^2 - x^2$$
$$y = \pm\sqrt{r^2 - x^2}$$

The equation represents two semicircles. The upper semicircle has the equation $y = \sqrt{r^2 - x^2}$ and the lower semicircle has the equation $y = -\sqrt{r^2 - x^2}$.

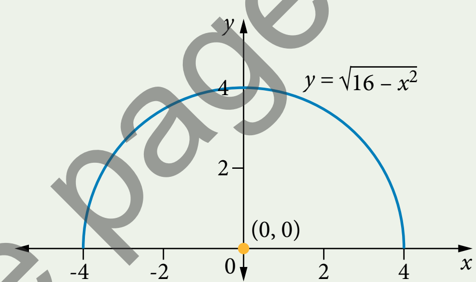


When graphed together, the full circle will be formed.

16 Sketching semicircles centred at the origin

Consider the semicircle with rule $y = \sqrt{16 - x^2}$.

(a) Sketch the graph.

THINKING	WORKING
1 Write the equation in the form $y = \pm\sqrt{r^2 - x^2}$.	$y = \sqrt{16 - x^2}$ $y = \sqrt{4^2 - x^2}$
2 From the equation, identify the value for r and the centre of the circle. Is this the upper or lower semicircle?	The radius is 4. This is the upper semicircle of a circle with centre at the origin.
3 Use the radius to determine the axes intercepts.	The coordinates of the x-intercepts are (-4, 0) and (4, 0). The coordinates of the y-intercept are (0, 4).
4 Sketch the graph of the function showing the centre and axes intercepts.	

(b) Write the domain and range using interval notation.

Write the domain and range of the function using interval notation.	Domain: [-4, 4] Range: [0, 4]

Semicircles centred at (a, b)

Transposing the general equation of a circle, centred at the (a, b) with radius r, to make y the subject gives:

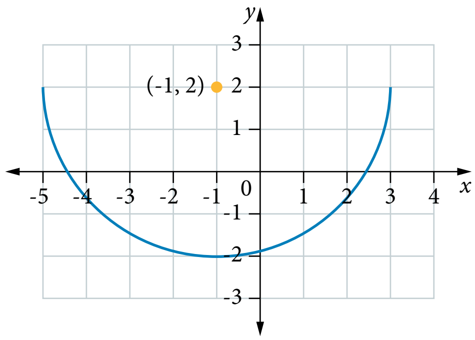
$$(x-a)^2 + (y-b)^2 = r^2$$
$$(y-b)^2 = r^2 - (x-a)^2$$
$$y - b = \pm\sqrt{r^2 - (x-a)^2}$$
$$y = b \pm \sqrt{r^2 - (x-a)^2}$$

The equation represents two semicircles. The upper semicircle has the equation $y = b + \sqrt{r^2 - (x-a)^2}$ and the lower semicircle has the equation $y = b - \sqrt{r^2 - (x-a)^2}$.

Semicircles centred: (0, 0):	Semicircles centred: (a, b):
Upper semicircle: $y = \sqrt{r^2 - x^2}$	Upper semicircle: $y = b + \sqrt{r^2 - (x-a)^2}$
Lower semicircle: $y = -\sqrt{r^2 - x^2}$	Lower semicircle: $y = b - \sqrt{r^2 - (x-a)^2}$

17 Equations of a semicircles with centre (a, b)

Determine the equation of the semicircle on the right.



THINKING	WORKING
1 Identify the coordinates of the centre. Determine the length of the radius r.	The coordinates of the centre are (-1, 2). The length of the radius is 4 units.
2 Identify which semicircle is shown.	This is the lower semicircle.
3 Write the equation in the form $y = b \pm \sqrt{r^2 - (x-a)^2}$.	$y = 2 - \sqrt{4^2 - (x+1)^2}$ $= 2 - \sqrt{16 - (x+1)^2}$
4 Check the reasonableness of the equation.	A circle with (-1, 2) and $r = 4$: $(x+1)^2 + (y-2)^2 = 4^2$ $(y-2)^2 = 16 - (x+1)^2$ $y - 2 = \pm\sqrt{16 - (x+1)^2}$ $y = 2 \pm \sqrt{16 - (x+1)^2}$ The graph is the lower half of the circle: $y = 2 - \sqrt{16 - (x+1)^2}$

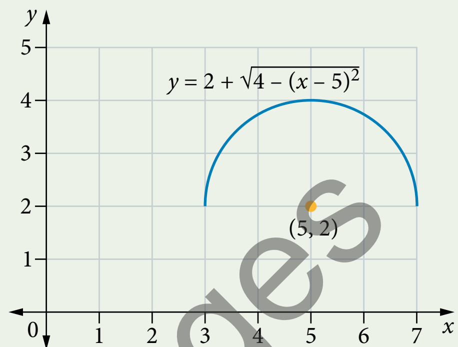
Piece-wise functions with semicircles

Semicircles can form part of a piece-wise function. Even though semicircles have a finite domain, it is usual to specify the domain within the rule for a piece-wise function.

18 Create a continuous piece-wise function involving a semicircle

Determine the values of the parameters so that the piece-wise function is continuous.

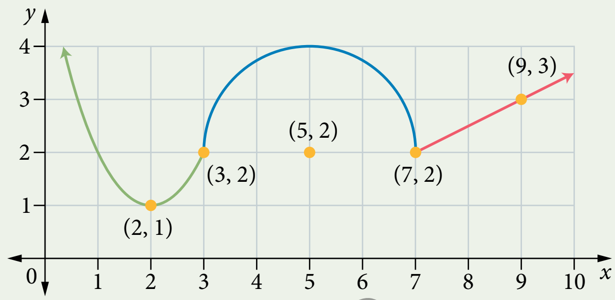
$$y = \begin{cases} a(x-2)^2 + 1, x < 3 \\ 2 + \sqrt{4 - (x-5)^2}, 3 \le x \le 7 \\ \dfrac{1}{2}x + c, x > 7 \end{cases}$$

THINKING	WORKING
1 Graph the semicircle, because the equation is known. Use $y = b + \sqrt{r^2 - (x-a)^2}$ with centre (a,b) and radius r.	Semicircle: $y = 2 + \sqrt{4-(x-5)^2}, 3 \le x \le 7$ $= 2 + \sqrt{2^2-(x-5)^2}, 3 \le x \le 7$ The coordinates of the centre are (5, 2). The length of the radius is 2 units. The semicircle is the upper half of a circle. 
2 Equate the expression for the quadratic and semicircle. Substitute the required value for x and hence solve for a.	Parabola and semicircle meet at $x = 3$: $a(x-2)^2 + 1 = 2 + \sqrt{4-(x-5)^2}$ $a(3-2)^2 + 1 = 2 + \sqrt{4-(3-5)^2}$ $a + 1 = 2 + 0$ $a = 1$
3 Determine the equation for the parabola.	The equation of the parabola is $y = (x-2)^2 + 1$
4 Specify any points needed to assist a sketch.	Turning point: (2, 1)
5 Equate the expression for the straight line and semicircle. Substitute the required value for x and solve for c.	Line and semicircle meet at $x = 7$: $\frac{1}{2}x + c = 2 + \sqrt{4-(x-5)^2}$ $\frac{1}{2} \times 7 + c = 2 + \sqrt{4-(7-5)^2}$ $\frac{7}{2} + c = 2 + 0$ $c = -\frac{3}{2}$
6 Determine the equation for the line.	The equation of the line is $y = \frac{1}{2}x - \frac{3}{2}$
7 Specify another point to assist a sketch.	For $x = 9$: $y = \frac{1}{2} \times 9 - \frac{3}{2}$ $= 3$ Another point is (9, 3).

8 Write the equation of the piece-wise function.

$$y = \begin{cases} (x-2)^2 + 1, x < 3 \\ 2 + \sqrt{4 - (x-5)^2}, 3 \le x \le 7 \\ \frac{1}{2}x - \frac{3}{2}, x > 7 \end{cases}$$

9 Sketch the graph to support your answers.



EXERCISE 4.3 Circles

Worked Example

1 Consider the relation $x^2 + y^2 = 4$ and answer the following questions. 11

(a) Sketch the graph of $x^2 + y^2 = 4$ showing the axes intercepts in coordinate form.

(b) Write the domain and range using interval notation.

2 (a) A point that lies on the circle with equation $(x-5)^2 + (y+2)^2 = 36$ is:

A (5, 2) B (-5, 2) C (5, 4) D (-5, -4)

(b) Explain the common error made by a student who thought the answer was (5, -2).

3 Determine the position of the following points in relation to the circle $x^2 + y^2 = 16$. 12

(a) (-1, 3) (b) (1.8, 2.5) (c) (-3.5, -2.1)

4 Determine the position of the following points in relation to a given circle. 14

(a) $(x-3)^2 + (y-1)^2 = 9$; (2, -1)

(b) $x^2 + (y+4)^2 = 16$; (-3, -3)

(c) $(x+2)^2 + y^2 = 16$; (-2, -4)

(d) $(x+3)^2 + (y-4)^2 = 14$; (-3, 5)

(e) $(x-7)^2 + (y+1)^2 = 21$; (6, 7)

(f) $(x+1)^2 + (y+1)^2 = 10$; (-4, 3)

5 A garden is laid out in a grid system with positions determined by pacing out distances in metres in two directions at right angles from the corner of the garden.
A petunia is growing at position (1, 3). Determine whether the petunia will receive water from a rotating sprinkler with a reach of 3 m, if the sprinkler is placed in each of the following positions.

(a) (6, 6) (b) (4, 5) (c) (2, 2)

6 The equation of the circle with centre (-2, -4) and radius $\sqrt{2}$ is:

A $(x-2)^2 + (y-4)^2 = \sqrt{2}$

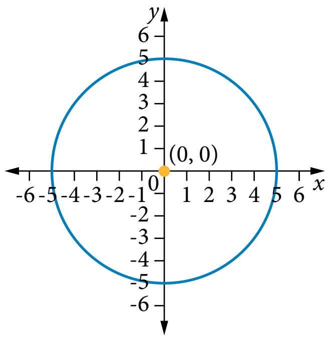
B $(x+2)^2 + (y+4)^2 = \sqrt{2}$

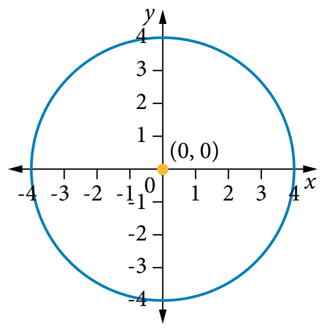
C $(x-2)^2 + (y-4)^2 = 2$

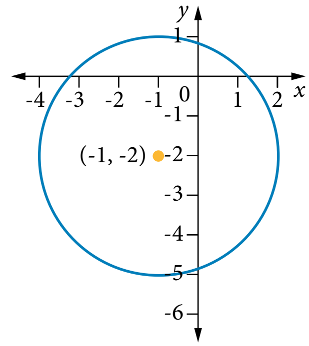
D $(x+2)^2 + (y+4)^2 = 2$

7 Determine the domain and range of the relation $(x+1)^2+(y-2)^2=1$.

8 Determine the equations of the following circles.

(a)


(b)


(c)


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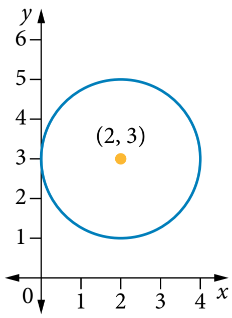
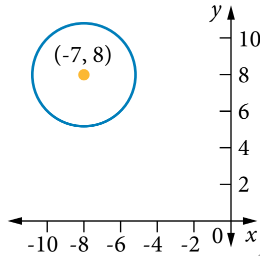
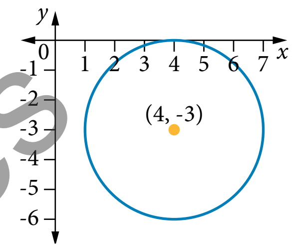
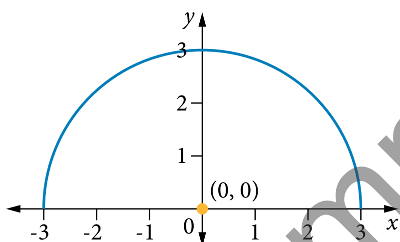
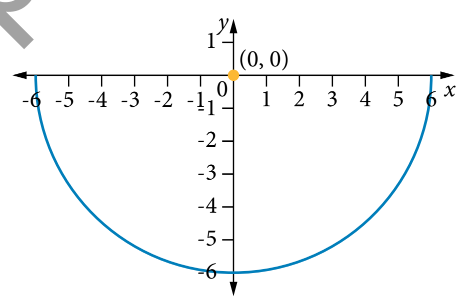


(e)


(f)


9 Determine the equations of the following semicircles.

(a)


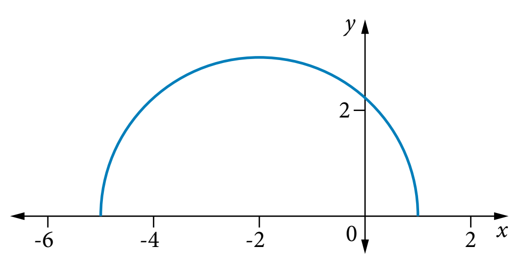
(b)


Worked Example

16 10 Sketch the graph of each semicircle. Identify the domain and range of each function.

(a) $y=\sqrt{25-x^2}$ (b) $y=-\sqrt{4-x^2}$ (c) $y=\sqrt{10-x^2}$ (d) $y=-\sqrt{20-x^2}$

17 11 Determine the equation of the semicircle shown.



15 12 For each of the following relations:

(i) write the following relations in the form $(x-a)^2+(y-b)^2=r^2$

(ii) determine the centre and radius of the circle

(iii) write the domain and range of the relation, using interval notation.

(a) $x^2+y^2+2y=0$ (b) $x^2+2x+y^2=3$

(c) $x^2+4x+y^2-2y=-4$ (d) $x^2-8x+y^2+6y=0$

Worked Example

13 Consider the relation with equation $(x+2)^2+(y-3)^2=16$. 13

(a) Determine the centre and radius of the circle.

(b) Determine, in exact form, the axes intercepts and then sketch the graph.

(c) Write the domain and range, using inequality symbols.

14 Graph the relation with equation $x^2+y^2+2x+4y=31$ showing all relevant features. State the domain and range.

15 Consider the relation with equation $(x-3)^2+(y+1)^2=10$.

(a) Determine the centre and exact radius of the circle.

(b) Determine the axes intercepts and then sketch the graph.

(c) Write the domain and range in exact form, using interval notation.

16 Consider the function with equation $y=-\sqrt{16-(x+1)^2}+2$.

(a) Determine the coordinates of the centre and the length of the radius of the semicircle.

(b) Determine the axes intercepts.

(c) Sketch the curve of the function, showing the coordinates for the endpoints and intercepts.

(d) Write the domain and range of the function using interval notation.

17 Consider the relation with equation $y=\sqrt{9-(x-1)^2}-3$.

(a) Identify the centre and radius of the function.

(b) Determine the axes intercepts.

(c) Sketch the curve of the function, showing the coordinates for the endpoints and intercepts.

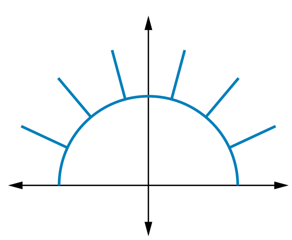
(d) Write the domain and range of the function using interval notation.

18 Determine the rule for the continuous piece-wise function and justify your answer with a graph. 18

$$f(x)=\begin{cases} a(x+4)^2+3, & x<-2 \\ \sqrt{4-x^2}, & -2\le x\le 2 \\ c-x, & x>2 \end{cases}$$

19 Consider the functions $g(x)=\sqrt{4-(x-2)^2}+2$, $h(x)=x+2$ and $P(x)=2x-6$. Using $g(x)$, $h(x)$ and $P(x)$, construct a continuous piece-wise function, $f(x)$ over the domain [-4, 8].

20 Consider the simplified diagram of a sunset, which can be produced by combining the graphs of several functions. Use your knowledge of piece-wise functions to create this image with equations and restricted domains. Let the radius of the semicircle be 3 units and the length of the rays be 2 units. The rays need not be equally spaced, but make sure your diagram is symmetrical about the vertical axis. Assume each ray is part of a line through the origin.

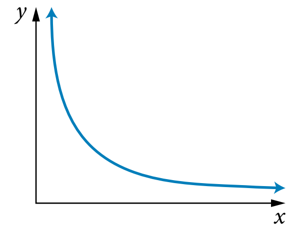


21 A rectangular piece of fabric, 18 cm by 24 cm, is to be decorated with scalloped edges, with each scallop consisting of a semicircle of diameter 6 cm. Write a list of rules to model the entire boundary of the finished article.

4 Summary

Inverse proportion

The algebraic relationship describing inverse proportion is of the form $y = \frac{k}{x}$, where the variables are x and y with k being a constant.

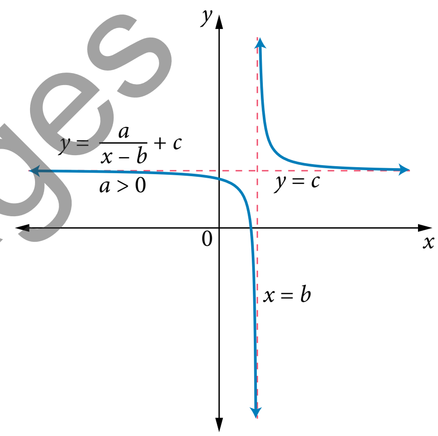


The graph of $y = \frac{k}{x}$, where $k > 0$, lies entirely within the first quadrant.

The graph approaches, but does not touch, the axes.

Graphs of rectangular hyperbolas

The graph of $y = \frac{a}{x-b} + c$ is called a rectangular hyperbola.



The vertical asymptote is $x = b$.

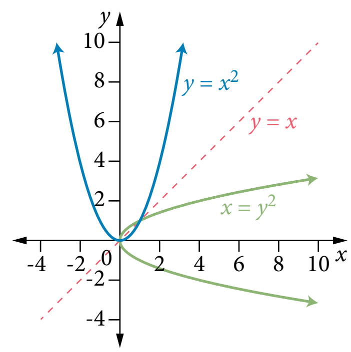
The horizontal asymptote is $y = c$.

The domain of the hyperbola is $\mathbb{R} \setminus \{b\}$.

The range of the hyperbola is $\mathbb{R} \setminus \{c\}$.

$f(x) = \frac{a}{(x-b)} + c$ can be represented by a linear function divided by another linear function in the form $f(x) = \frac{a + c(x-b)}{x-b}$.

The graph of $x = y^2$

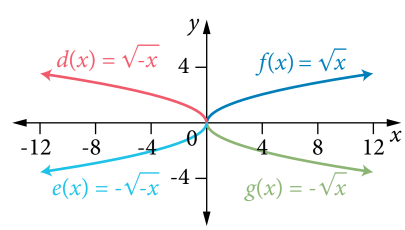


The relation $x = y^2$ is not a function. Its graph can be obtained from the function $y = x^2$ by reflecting in the line $y = x$.

The vertex of $x = y^2$ is $(0, 0)$.

The line of symmetry is the x-axis: $y = 0$.

The function $y = \sqrt{x}$



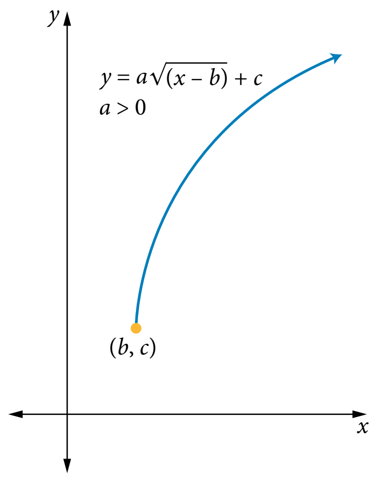
The function $f(x) = \sqrt{x}$ corresponds to the positive (and zero) part of the relation $x = y^2$. The range is also the set of non-negative numbers, $[0, \infty)$, or simply $y \geq 0$.

The function $g(x) = -\sqrt{x}$ corresponds to the negative (and zero) part of the relation $x = y^2$.
The graph of $g(x) = -\sqrt{x}$ is the reflection of the graph of $f(x) = \sqrt{x}$ in the x-axis.

The graph of the function $d(x) = \sqrt{-x}$ is the reflection of the graph of $f(x)$ in the y-axis and the graph of the function, $e(x) = -\sqrt{-x}$ is the reflection of the graph of $g(x)$ in the y-axis.

Graphs of square roots

The graph of $y = a\sqrt{(x-b)} + c$ or $y = a\sqrt{-(x-b)} + c$ is called a square root graph.



The vertex is (b, c).

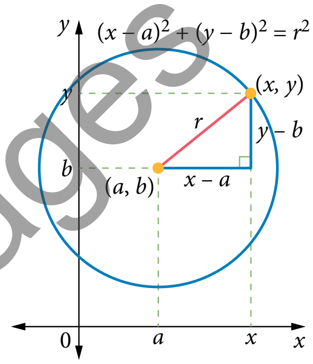
The domain of the square root graph is:

- $[b, \infty)$, where the coefficient of x is a positive number
- $(-\infty, b]$, where the coefficient of x is a negative number.

The range of the square root graph for $a > 0$ is $[c, \infty)$, and for $a < 0$ is $(-\infty, c]$.

Circles

The graph of the relation $x^2 + y^2 = r^2$ is a circle with centre $(0, 0)$ and radius r, where $r > 0$.



The graph of the relation $(x-a)^2 + (y-b)^2 = r^2$ is a circle.

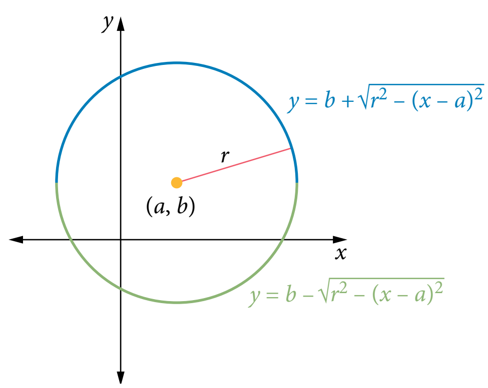
The centre is (a, b) and radius is r.

Domain: $[a - r, a + r]$

Range: $[b - r, b + r]$

Semicircles

The graph of $y = b + \sqrt{r^2 - (x-a)^2}$ is the upper semicircle of the general circle $(x-a)^2 + (y-b)^2 = r^2$:



Domain: $[a - r, a + r]$

Range: $[b, b + r]$

The graph of $y = b - \sqrt{r^2 - (x-a)^2}$ is the lower semicircle of the general circle $(x-a)^2 + (y-b)^2 = r^2$:

Domain: $[a - r, a + r]$

Range: $[b - r, b]$

Chapter review 4

Exercise 4.1

1 As a piston is pushed down into a cylinder, the volume decreases and the pressure increases. The pressure P (Pa) inside the cylinder is inversely proportional to the volume V (cm^3). When the volume in the piston is $50\,cm^3$ the pressure is 0.05 Pa. Determine the volume when the pressure is 0.001 Pa.

Exercise 4.1

2 Ash decided to test how long it would take a $5\,cm^3$ ice cube to melt at different room temperatures in his kitchen. The table on the right shows the times t (min) it took to fully melt the ice cube at temperature T°C.

T°C	12	24	30
t (min)	17	8.5	6.8

Estimate the time taken to melt a $5\,cm^3$ ice cube at a room temperature of 7°C. Answer correct to 1 decimal place.

Exercise 4.1

3 The speed S (m/s) at which an object flies for a fixed distance of 120 m varies with the time in seconds according to the formula $S = \frac{120}{t}$.

(a) Sketch the graph for the first 120 seconds of travel.

(b) Determine the speed, given that the object takes 60 seconds to complete its flight.

Exercise 4.1

4 Sketch the graph of $f(x) = \dfrac{2}{x-2} + 2$, clearly showing axes intercepts and asymptotes.

Exercise 4.1

5 Sketch the graph of $f(x) = -\dfrac{4}{x+1} + 8$, clearly showing all intercepts and asymptotes.

Exercise 4.1

6 Write each expression in the form $\dfrac{a}{x-b} + c$.

(a) $\dfrac{3x-1}{x+2}$ **(b)** $\dfrac{2-x}{x+3}$ **(c)** $\dfrac{x}{x+4}$ **(d)** $\dfrac{3x}{x+5}$

Exercise 4.1

7 The general gas equation states that $PV = kT$, where P (pascals) is the pressure of the gas, V (m^3) is the volume of a container, T is the temperature in kelvin and k is a constant representing a fixed amount of gas. Draw a sketch for each of the following situations.

(a) The volume is fixed as a constant and the pressure varies, depending on the temperature.

(b) The temperature is fixed as a constant and the pressure depends on the volume.

(c) The temperature is fixed as a constant and the volume depends on the pressure.

Exercise 4.2

8 Sketch the graph of $f(x) = 2\sqrt{4-x} - 6$ with clearly labelled intercepts and vertex.

Exercise 4.2

9 Sketch the graph of each relation using technology. Hence, or otherwise, label the coordinates of the vertex, the equation of the line of symmetry and an additional point.

(a) $x = \dfrac{1}{5}y^2$ **(b)** $x = y^2 + 1$

Exercise 4.2

10 The speed of tsunami in the open ocean is given by the formula $c = \sqrt{gH}$, where c is the speed in ms^{-1}, $g = 9.8\,ms^{-2}$ and H is the ocean depth in metres.

(a) Sketch a graph of c as a function of H for depths of at least 5000 m.

(b) Determine the speed of a tsunami at the common ocean depth of 4000 m. Write your answer to the nearest whole number. Verify your answer with a graph.

11 The function $f(x) = \sqrt{x}$ is changed according to the following transformations. Write the equation for each. Exercise 4.2

(a) For $g(x)$, $f(x)$ is reflected in the x-axis, translated 2 units left and 1 unit down.

(b) For $h(x)$, $f(x)$ is dilated by factor 2 parallel to the y-axis, reflected in the x-axis, translated 4 units left and 1 unit down.

(c) For $p(x)$, $f(x)$ is dilated by factor 4 parallel to the y-axis, reflected in the x-axis, translated 2 units right and 5 units down.

(d) For q(x), $f(x)$ is dilated by factor 2 parallel to the y-axis, reflected in the x-axis, translated 3 units right and 6 units up.

12 The transformations required to produce the graph of $y = -3\sqrt{x+1} - 2$ from $y = \sqrt{x}$ are: Exercise 4.2

A dilation of factor 3 parallel to the y-axis, reflection in the x-axis, translation 2 units up and 1 unit left

B dilation of factor 3 parallel to the y-axis, reflection in the x-axis, translation 2 units down and 1 unit left

C dilation of factor 3 parallel to the y-axis, translation 2 units down and 1 unit left

D dilation of factor 3 parallel to the y-axis, reflection in the x-axis, translation 1 unit down and 2 units left

13 Consider $(x-2)^2 + (y-4)^2 = 16$. Exercise 4.3

(a) Sketch the graph of the relation, labelling the axes intercepts.

(b) Determine the domain and range.

14 Consider $y + 1 = \sqrt{4-(x+2)^2}$. Exercise 4.3

(a) Sketch the graph of the relation, labelling the axes intercepts.

(b) Determine the domain and range.

15 Determine whether the following points lie within the circle $x^2 + y^2 = 14$. Exercise 4.3

(a) (2.4, 1.3)

(b) (-2.6, -2.7)

16 Determine whether each point is within or outside the given circle. Exercise 4.3

(a) $(x+5)^2 + (y-4)^2 = 16$; (-7, 5)

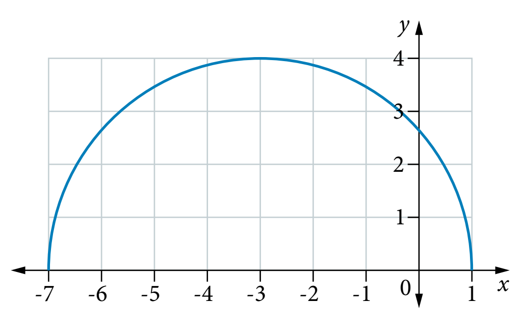
(b) $(x+5)^2 + y^2 = 8$; (-4, -3)

17 Sketch the graph of each semicircle, stating the domain and range of each function. Exercise 4.3

(a) $y = -\sqrt{49 - x^2}$

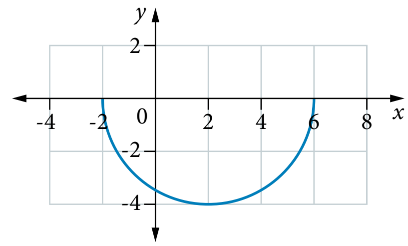
(b) $y = \sqrt{13 - x^2}$

18 Determine the equation of the semicircle shown. Exercise 4.3



Exercise 4.3 **19** Determine the equation of the circle with radius 9 units and centre (-1, 2).

Exercise 4.3 **20** Determine the equation of the semicircle shown.



Exercise 4.1 **21** Consider the graph of $f(x) = -\dfrac{2}{4-x} + 4$.

(a) The coordinates of the x- and y-intercepts are, respectively:

A $\left(3\frac{1}{2}, 0\right), \left(0, 3\frac{1}{2}\right)$

B $\left(3\frac{1}{2}, 0\right), \left(0, 4\frac{1}{2}\right)$

C $\left(4\frac{1}{2}, 0\right), \left(0, 4\frac{1}{2}\right)$

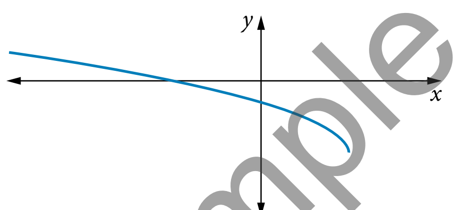
D $\left(4\frac{1}{2}, 0\right), (0, 4)$

(b) Sketch a graph of the function.

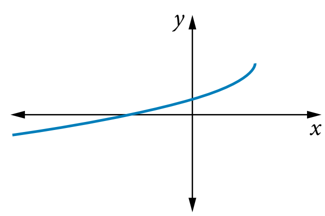
Exercise 4.2 **22** Consider the function $f(x) = -\sqrt{2-x} - 2$.

(a) Choose the graph that most accurately represents the function.

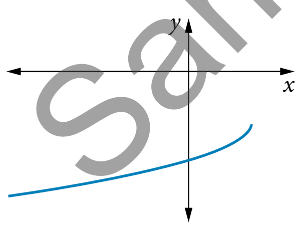
A



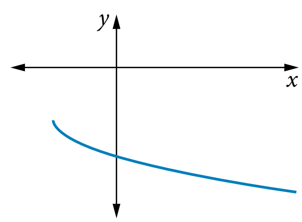
B



C



D



(b) Identify the coordinates of the vertex and determine any intercepts on the axes.

Exercise 4.1 **23** The graph $f(x) = \frac{1}{x}$ is reflected in the x-axis, dilated by a factor of 2 parallel to the y-axis, translated 4 units right and then translated 2 units down to produce $g(x)$.

(a) Determine the equation for $g(x)$.

(b) Sketch the graph of $f(x)$ and $g(x)$ on the same set of axes.

Exercise 4.1 **24** Sketch the graph of $y = \dfrac{2x+3}{x-1}$, showing all key points.

Exercise 4.2 **25** Sketch the graph of $f(x) = 2\sqrt{x+2} - 4$, showing the axes intercepts in exact form.

26 Sketch the piece-wise function $f(x)$ and write the domain and range in interval form. Exercise 4.2

$$f(x)=\begin{cases} 1-x, & x<-4 \\ \sqrt{x+4}+5, & -4\le x<5 \\ x+3, & x\ge 5 \end{cases}$$

27 Sketch the graphs of the following, showing all significant points labelled in exact form. Exercise 4.2

(a) $y=2\sqrt{2x+3}-1$

(b) $y=1-4\sqrt{3-2x}$

28 Consider the relation $x^2-2x+y^2-4y-4=0$. Exercise 4.3

(a) Write the relation in the form $(x-h)^2+(y-k)^2=r^2$ and hence identify the centre and radius of the circle.

(b) Sketch the circle, labelling the intercepts in exact form.

(c) A circle can be split into an upper and lower semicircle. Give the equation of each semicircle, clearly stating which represents the upper semicircle.

(d) If the original circle was translated 2 units right and 3 units down, identify the equation of the circle and determine the axes intercepts in exact form.

29 A graph of the form $y=\frac{ax+b}{cx+d}$ has both domain and range of $\mathbb{R}\setminus\{3\}$ and passes through the point (1, 5.5). Write the equation of the graph in the given form. Sketch the graph, showing exact values of any intercepts on the axes. Exercise 4.1

30 The hyperbola $f(x)$ with asymptotes at $x=1$ and $y=2$ passes through the points (2, 3) and (0, 1). Exercise 4.2

(a) Determine the rule of the graph and sketch the hyperbola, clearly labelling the asymptotes, intercepts and the point (2, 3).

(b) $f(x)$ is dilated by factor 2 from the x-axis and translated 3 units units left and 4 units down to produce $g(x)$. Determine the equation for $g(x)$ and sketch it showing all the major features.

(c) $g(x)$ is reflected in the x-axis to produce $h(x)$. Determine the equation for $h(x)$ and the axes intercepts.

(d) $g(x)$ is reflected in the y-axis to produce $p(x)$. Determine the equation for $p(x)$ and the axes intercepts.

31 The piece-wise function $f(x)$ is defined as follows: Exercise 4.3

$$f(x)=\begin{cases} x+2, & x\le 1 \\ g(x), & 1<x\le 7 \\ h(x), & x>7 \end{cases}$$

If $g(x)$ is a semicircle and $h(x)=c-x$, calculate the value of c and determine the equations of two semicircles that would make $f(x)$ a continuous function. Graph $f(x)$ for both cases.